DOCUMENT RESUME

ED 127 198 SE 021 245

TITLE National Assessment of Educational Progress. The First National Assessment of Mathematics: An Overview.
INSTITUTION Education Commission of the States, Denver, Colo. National Assessment of Educational Progress.
SPONS AGENCY National Center for Education Statistics (DHEW), Washington, D.C.
REPORT NO NAEP-04-MA-00
PUB DATE Oct 75
NOTE 64p.; Contains small print in Figures
AVAILABLE FROM Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (Mathematics Report Number 04-MA-00, $1.25)

EDRS PRICE MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS *Educational Assessment; *Educational Research; Elementary Education; Evaluation; Higher Education; Mathematical Concepts; *Mathematics; Mathematics Education; *National Surveys; Secondary Education; *Student Ability
IDENTIFIERS NAEP; *National Assessment of Educational Progress; Research Reports

ABSTRACT

This report summarizes the results of a nationwide survey of the mathematical ability of young Americans at four age levels: 9-year-olds, 13-year-olds, 17-year-olds, and young adults ages 26-35. The study was conducted during the 1972-73 school year by the National Assessment of Educational Progress (NAEP). The mathematics assessment included six major content areas: numbers and numeration, measurement, geometry, variables and relationships, probability and statistics, and consumer mathematics. Each chapter summarizes results for one content area and indicates trends in ability illustrated by results for selected exercises. Concepts from all content areas are usually introduced at the elementary level and are then reinforced and expanded at higher age levels. In addition to age levels, the assessment also provides results for the following groups within the national population: sex, race, region of the country, level of parental education, and size and type of community. Results for the different population groups are not given for each content area but are discussed in the data summary from all content areas. (Author/MH)

MATHEMATICS
AN OVERVIEW

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THE FIRST NATIONAL ASSESSMENT OF MATHEMATICS:

An Overview

Mathematics Report No. 04-MA-00

October 1975

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402

NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

Roy H. Forbes
Director

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Associate Director

This publication was prepared and produced pursuant to agreements with the National Center for Education Statistics, Office of the Assistant Secretary for Education, Department of Health, Education, and Welfare, with additional funds from the Carnegie Corporation of New York and the Ford Foundation's Fund for the Advancement of Education. The statements and views expressed herein do not necessarily reflect the position and policy of the Office of the Assistant Secretary for Education or other grantors, but are solely the responsibility of the National Assessment of Educational Progress, a project of the Education Commission of the States.

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FOREWORD

The National Assessment of Educational Progress (NAEP) is an information-gathering project that surveys the educational attainments of 9-year-olds, 13-year-olds, 17-year-olds and adults (ages 26–35) in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure educational change.

Each assessment is the product of several years work by a great many educators, scholars and lay persons from all over the country. Initially, these people design objectives for each area, proposing specific goals that they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are then given to exercise (item) writers, whose task it is to create measurement tools appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists and measurement experts, they are administered to probability samples from various age levels. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can generalize about the probable performance of all 9-year-olds in the nation.

National Assessment also publishes a general information yearbook that describes all major aspects of the Assessment's operation. The reader who desires more detailed information about how NAEP defines its groups, prepares and scores its exercises, designs its samples and analyzes and reports its results should consult the *General Information Yearbook*, *Report 03/04-GIY*.

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ACKNOWLEDGMENTS

Many people have made substantial contributions to the mathematics assessment, from the beginning of the National Assessment of Educational Progress (NAEP) in 1964 to this third report of findings in the area of mathematics. Unfortunately, it is not possible to acknowledge them all here, and an apology is due to those whose names have been omitted.

The original preparation of the objectives and exercises in the area of mathematics was handled by the Educational Testing Service (ETS) and The Psychological Corporation. These materials were reviewed by dozens of consultants, including mathematicians, mathematics educators and interested lay persons, under the general monitoring of the National Assessment staff. Special mention must be made of three individuals and their contributions to the developmental phases: Emil Berger of the St. Paul Public Schools (Minnesota) for his assistance in finalizing the objectives and efforts in developing and field testing exercises, and Dale Foreman of Westinghouse Learning Corporation and Todd Rogers of the University of British Columbia (former NAEP staff members) for their efforts in developing the individually administered mathematics exercises.

The administration of the mathematics assessment was conducted by the Research Triangle Institute (RTI) and the Measurement Research Center (MRC). Scoring and processing were carried out by MRC and by the NAEP staff. Louise Diana of MRC and Fred Schippert of the Detroit Public Schools (Michigan) provided invaluable assistance in developing and refining the categories used to score the exercises. James Wilson of the University of Georgia (Athens) and Robert Reys of the University of Missouri (Columbia) were extremely helpful in suggesting analysis schemes.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must be given to the following people and departments: Hugh Cobb, James Damon and Eric Morgan, Data Processing Department; Ava Powell, Research Assistant, Research and Analysis Department; and Marci Reser and Eileen Wollam, Production Assistants, Utilization/Applications Department. Technical analysis for this report was planned and supervised by Wayne Martin; the report was written by Barbara Ward.

Special thanks must also go to J. Stanley Ahmann, who directed the NAEP program throughout the period in which this information was gathered and reported.

Roy H. Forbes
Project Director

INTRODUCTION

During the 1972—73 school year, the National Assessment of Educational Progress (NAEP) conducted a nationwide survey of the mathematical ability of young Americans at four age levels — 9-year-olds, 13-year-olds, 17-year-olds and young adults ages 26—35.[1] The mathematics assessment included six major content areas: numbers and numeration, measurement, geometry, variables and relationships, probability and statistics and consumer mathematics. These content areas covered most of the topics ordinarily taught in general mathematics classes and some of the concepts encountered in elementary algebra. Very few exercises required mathematics courses beyond elementary algebra.

This report provides an overview of results for the entire mathematics assessment. The intent of this volume is not to provide an exercise-by-exercise review of results, but to indicate trends in ability illustrated by results for selected exercises.

Each chapter summarizes results for one content area. The arrangement of the content areas in this report does not signify their relative importance or the order in which they commonly appear in a school curriculum. Concepts from all the content areas are usually introduced at the elementary level and are then reinforced and expanded as students mature. For example, in geometry, 9-year-olds might be expected to name a rectangle and a triangle, while 17-year-olds and adults might be asked to apply the Pythagorean theorem.

[1]The reader desiring information about specific National Assessment data-collection procedures should consult the *General Information Yearbook, Report 03/04-GIY* (Washington, D.C. Government Printing Office, 1974)

The mathematics curriculum, like that of many other learning areas, underwent substantial revisions in the decade of the sixties. New terms were introduced; topics such as algebra and geometry were presented in the elementary grades, and the reasoning behind mathematical operations was explained to students. National Assessment took account of these revisions in planning its assessment and measured achievement in both "modern-math" and "traditional-math" topics and terminology. The exercises concerned with modern-math concepts indicate the number of people familiar with such concepts, but the assessment data are not intended to support comparisons of curricula or teaching methods.

In designing the measurement instruments for the mathematics assessment, National Assessment included some variations from the typical paper-and-pencil mathematics test. Although the majority of the exercises were printed in booklets and administered to groups of 8 to 12 people, some exercises were given on an individual basis. "Individual" exercises were used to elicit responses that would be difficult to observe in a group situation. In mathematics, these exercises were often used to observe the process that a person used to solve a problem.

Many of the mathematics exercises were open-ended (fill in the blank) rather than multiple-choice. The responses to the open-ended exercises were tabulated in various scoring categories. These categories revealed percentages of people making particular errors and thus provided some diagnostic information about common mathematical mistakes. Responses that could not be placed in any of the error categories were placed in a category called "other unacceptable." Respondents were instructed to write the words "I don't

know" on the answer line or to fill in the oval beside the "I don't know" choice if they felt they did not know the answer to a problem.

Approximately one-half of the exercises administered in a learning area for a given assessment year are released or made available for publication. The unreleased exercises are kept secure so that they can be reassessed in a future assessment to measure changes in ability levels. In this report, results for both released and unreleased exercises are discussed; however, actual exercise text appears only for released exercises.

National Assessment reports national percentages of success for 9-year-olds, 13-year-olds, 17-year-olds and young adults. The Assessment also provides results for the following groups within the national population: sex, race, region of the country, level of parental education and size and type of community. Results for the different population groups are not given for each content area but are discussed in the summary of data from all content areas (Chapter 7).

This report presents a general picture of performance for the different age levels and population groups for the entire mathematics assessment. Readers should consult the mathematics selective reports and the mathematics technical volumes for additional information.

CHAPTER 1

NUMBERS AND NUMERATION

Numbers provide a means for quantifying objects in a uniform fashion. Numeration is the system used for naming and organizing numbers. For example, although different words and symbols can be used to identify the concept "three," the idea of threeness remains the same. Different systems of numbers having specific properties have also been defined. The properties of a number system help to explain the reasons for the "rules" of mathematical operations in that particular system.

The first section of this chapter concerns numeral systems; the second, number systems and their properties; the third, set theory. The fourth section describes computation and problem-solving skills, which demand the application of the concepts included in the first three sections. Results for representative exercises measuring abilities with numbers and numeration are discussed in this chapter, and results for all exercises in the numbers and numeration content area are summarized at the conclusion of the chapter.

Numeral Systems

Different methods of "naming" numbers have been used in different cultures over the years. Our numeral system is a positional system based upon multiples of 10. This means that the position of each numeral affects its value. For example, the numeral 1 equals 1 when located in the 1s column; the same numeral equals 10 when placed in the 10s column. After students learn the numerals from 1 to 9, they can generate the symbols for larger numerals using the concepts of place value.

Several exercises assessed 9-year-olds' ability to deal with place value. Seventy-five percent of the nation's 9-year-olds correctly selected 6 when asked, "Which digit is in the 10s place in 4,263?" Similarly, 74% of the 9-year-olds identified 700 + 60 + 2 as the proper expansion of 762. A larger percentage, 87% at age 9, wrote a three-digit number in Arabic numerals after hearing it expressed verbally.

Respondents at the three upper age levels were requested to use numerals with a base other than 10. Forty-two percent of the 13-year-olds, 50% of the 17-year-olds and 38% of the adults chose the correct meaning of 25 in a base-9 system. Changing a base-10 number to a base-5 number proved more difficult, with 18% of the 13-year-olds, 14% of the 17-year-olds and 6% of the adults giving the correct answer. In this instance, percentages of success were highest at age 13 and dropped successively at the two upper age levels.

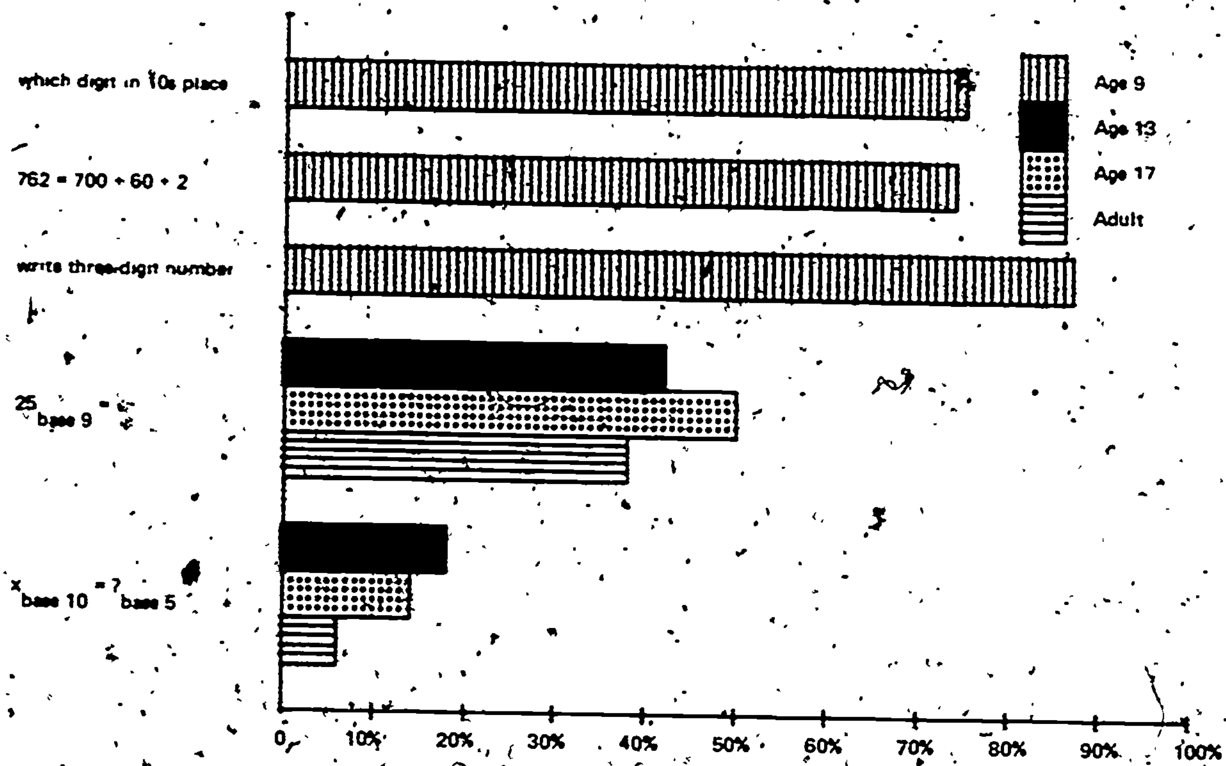
The exercises in this section indicate that approximately 75% of the 9-year-olds were able to use concepts of place value. Familiarity with bases other than 10 was not widespread, particularly at the adult level. Figure 1 provides a quick comparison of results on the exercises discussed in this section.

Number Systems and Their Properties

Whole Numbers

After students learn the principles of the decimal-place-value numeral system, they begin to study the properties of number sys-

FIGURE 1. Results for Selected Exercises: Numeral Systems



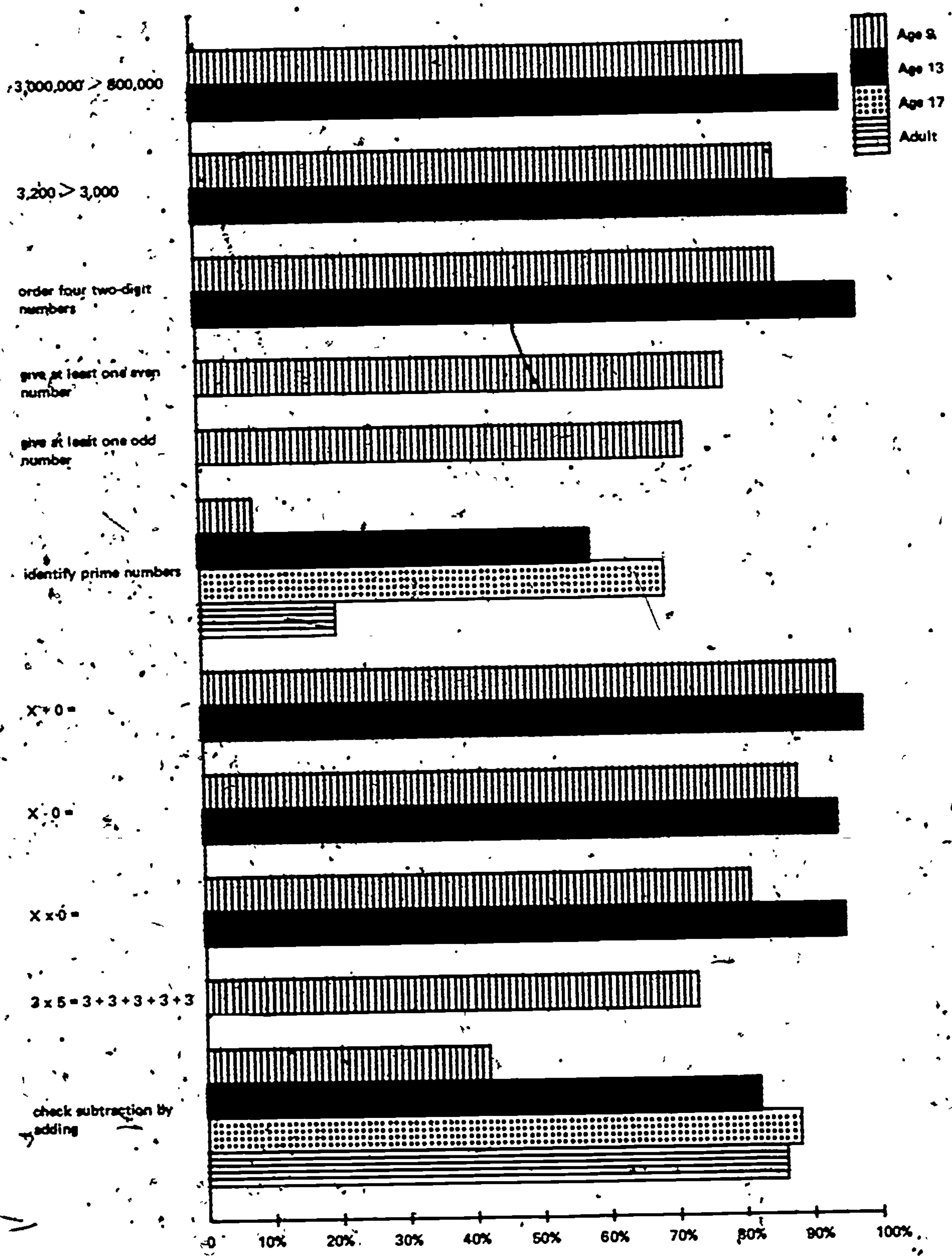
tems. The system of whole or counting numbers is the first system encountered. Some properties of this system include the method of ordering numbers and the existence of odd, even and prime numbers. Other properties concern operations, with whole numbers — for example, the closure, associative, commutative and distributive properties for addition and multiplication.

Over 80% of the 9-year-olds and over 95% of the 13-year-olds successfully answered three questions about number order. Eighty-two percent of the 9-year-olds and 96% of the 13-year-olds knew that 3,000,000 was greater than 800,000; 86% at age 9 and 97% at age 13 knew that 3,200 was greater than 3,000 and 86% of the 9-year-olds and 98% of the 13-year-olds arranged four two-digit numbers in the proper order.

Several exercises dealt with identification of odd, even and prime numbers. Seventy-eight percent of the 9-year-olds were able to write at least one even number, and 72% could give at least one odd number. Identification of prime numbers (numbers evenly divisible only by themselves and one) was easier for 13- and 17-year-olds than for 9-year-olds and adults. Given a choice of four numbers, 8% at age 9, 58% at age 13, 69% at age 17 and 20% at adult correctly indicated the prime number.

Other assessment items concerned properties of operations with whole numbers. Most 9- and 13-year-olds were familiar with the additive and multiplicative properties of zero. Ninety-four percent of the 9-year-olds and 98% of the 13-year-olds successfully added zero and a given number; 88% at age 9 and 94% at age 13 correctly subtracted zero from

FIGURE 2. Results for Selected Exercises. Properties of Whole Numbers



a given number, and 81% at age 9 and 95% at age 13 were able to multiply a number by zero.

At age 9, 73% indicated that they understood that multiplication is a form of addition; answering correctly that 3×5 equals $3 + 3 + 3 + 3 + 3$. Over 80% at the upper three age levels (82% at age 13, 88% at age 17 and 86% at adult) and 42% at age 9 knew how to use addition to check a subtraction problem, showing some knowledge of the relationship between addition and subtraction.

Figure 2 shows results for the exercises discussed above. Percentages of success were high for the order properties and identity properties of whole numbers and lower for identification of odd, even and prime numbers.

Rational Numbers

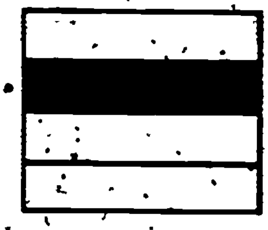
Another number system introduced in the elementary grades is the system of rational numbers. Rational numbers are numbers of the form a/b where $b \neq 0$. These numbers can be expressed as fractions, terminating or repeating decimals or percents. At the 9-year-old level, students learn that fractions are parts of a whole; they begin to appreciate relative sizes of fractions, and they also learn how to represent fractions in numerical symbols. At the upper age levels, additional methods of representing rational numbers — decimals and percents — are introduced, and properties and operations for rational numbers are discussed.

Nine-year-olds answered several questions of the type shown in Exhibit 1.

Thirty-one percent of the 9-year-olds correctly identified 1/4 of a rectangle, 2/6 of a circle and 2/5 of a circle. Slightly more (37%) named 4/8 of a rectangle. However, only 18% correctly labeled 1/3 of a rectangle. Twenty-five percent of the 9-year-olds answered all of the first four examples acceptably; 60% did not answer any of the four items correctly.

EXHIBIT 1. 9-Year-Old Exercises About Representation of Fractions

What fractional part of the figure below is shaded?



ANSWER ____________

Sixty-five percent of the 13-year-olds and 81% of the 17-year-olds successfully selected the proper reduction of 47/5 (9 2/5). The 9-year-old percentage of success on this exercise was low (about 7%).

Respondents were also asked to apply the order property to fractional numbers. Nineteen percent of the 13-year-olds, 39% of the 17-year-olds and 36% of the adults knew that 3/16 was closer to 5/32 than to 1/4, 5/16 or 3/8. Fifty-six percent of the 13-year-olds and 83% of the 17-year-olds were successful when requested to identify the fraction between two given fractions.

Representation of rational numbers by decimals and percents was assessed at the three older age levels. The decimal .3333... was successfully translated as 1/3 by 12% at age 13, 41% at age 17 and 52% at adult. About two-fifths of the 13-year-olds and two-thirds of the 17-year-olds correctly gave the percent equivalent to 1/5 (20%).

In addition, respondents determined the proper order for several sets of decimals. Close to 85% of the 13-year-olds and over 90% of the 17-year-olds and adults correctly selected 5.0 as greater than 0.5, 0.05 and 0.005. Percentages of success were lower — 51% at age 13, 75% at age 17 and 74% at adult — for choosing 0.022 as being smaller than 2.002, 0.22 or 0.202. Figure 3 shows results for the selected exercises involving rational numbers.

FIGURE 3. Results for Selected Exercises: Rational Numbers

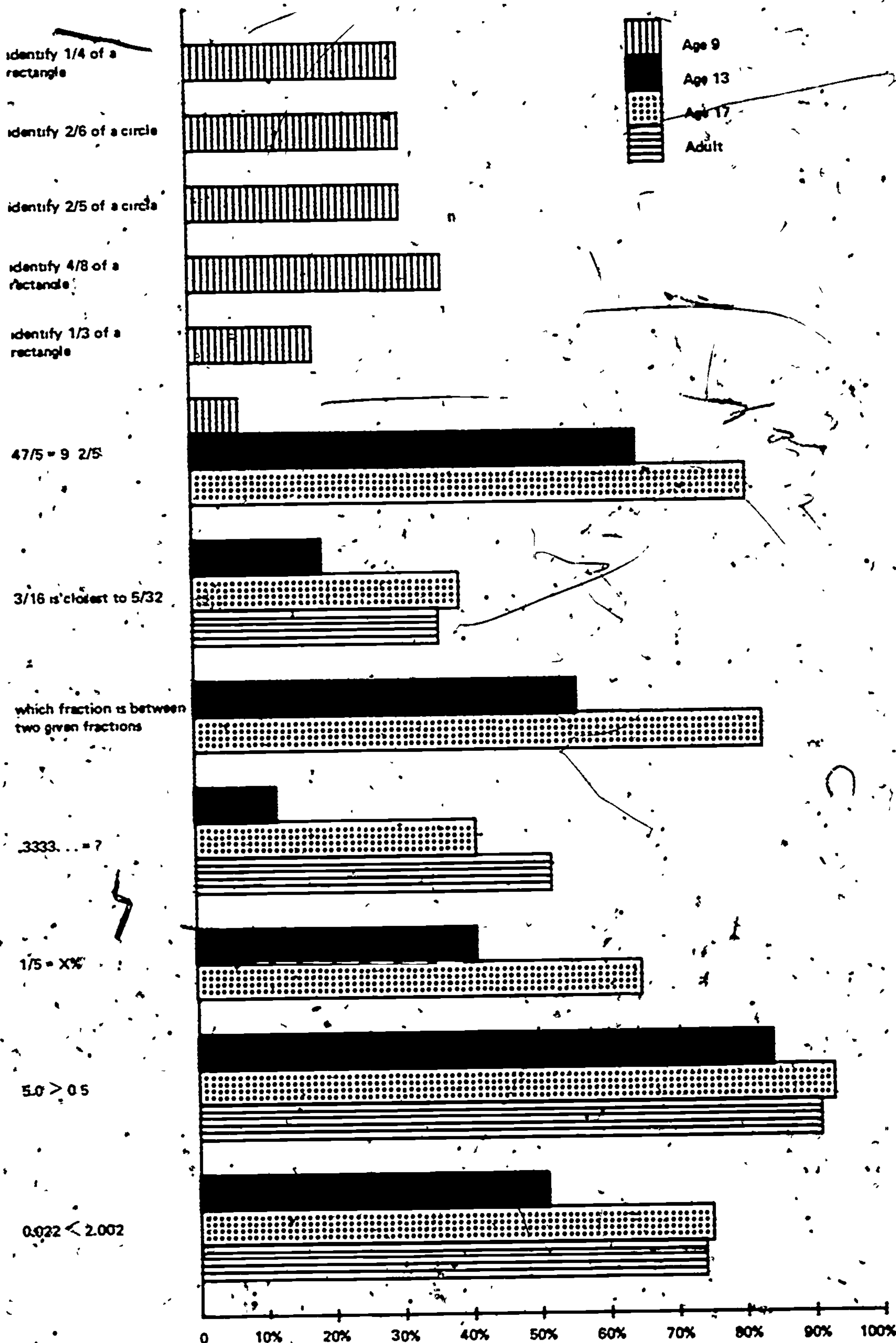
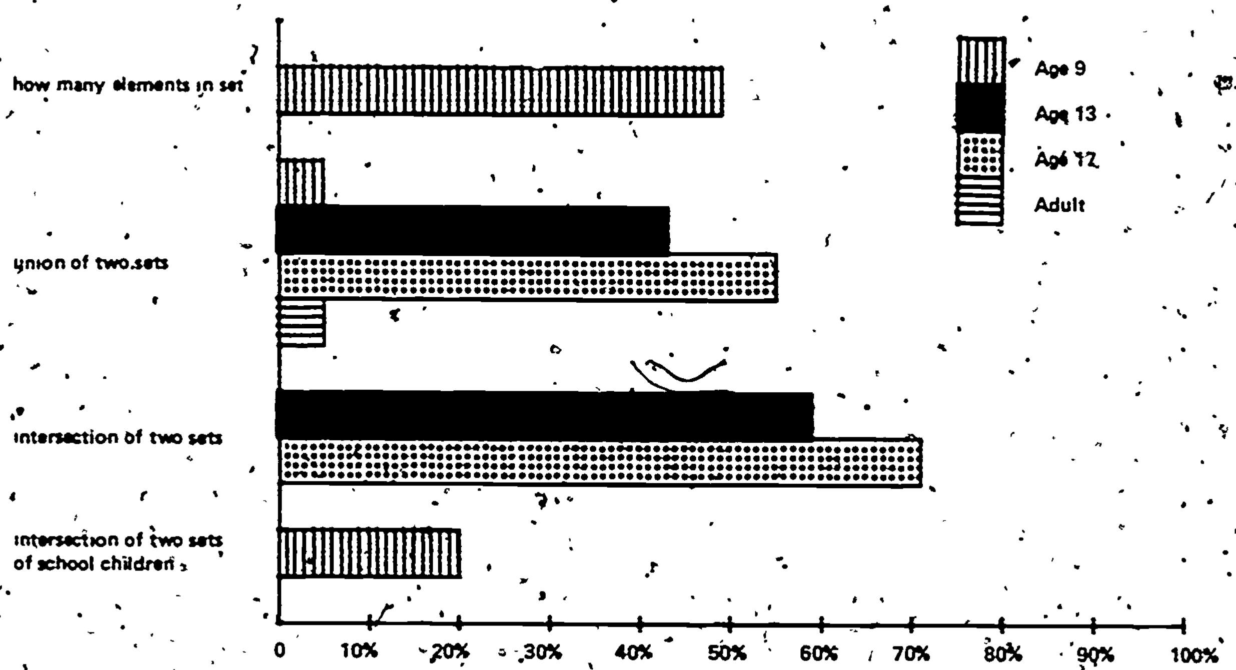


FIGURE 4. Results for Selected Exercises: Set Theory



Set Theory

Set theory provides a language for organizing and expressing mathematical concepts. Knowledge of the terminology and properties of sets is useful in understanding the properties of number systems and forms a foundation for learning more advanced mathematics, for example, topology, advanced algebra or calculus. Exercises about sets included in the mathematics assessment were focused upon set terminology and simple manipulations of sets.

Nearly half of the 9-year-olds correctly answered that there were four elements in the following set. {6, 3, 2, 7}. Forty-three percent at age 13 and 55% at age 17, but only 5% at ages 9 and adult, successfully listed the union of two sets. The intersection of two sets of numbers was correctly given by 59% of the 13-year-olds and 71% of the 17-year-olds. Approximately one out of five 9-year-olds properly stated the intersection of two sets of school children. Figure 4 presents results for the four age levels for these exercises about sets.

Operations With Real Numbers — Arithmetic Computation

Students may use the properties of number systems and the principles of set theory in learning to add, subtract, multiply and divide — first with whole numbers, later with rational numbers and integers. While developing computational skills, students also learn problem-solving techniques so that they can apply mathematics in various situations.

Results for the computation exercises in the mathematics assessment are summarized briefly in this overview, readers desiring more detailed data on results for mathematics computation should consult the report *Math Fundamentals. Selected Results From the First National Assessment of Mathematics, Report 04-MA-01.*

Respondents at all age levels solved four problems using each of the four computational operations with whole numbers. The problems and results for each age level appear in Table 1. The results show that 9-year-olds had difficulty with multiplication and divi-

TABLE 1. Whole-Number Computation

		Age 9	Age 13	Age 17	Adult
A	Add				
	38 +19				
	57*	[illegible]	94%	97%	97%
B	Subtract				
	36 -19				
	17*	55	89	92	92
C	Multiply				
	38 x9				
	342*	25	83	88	81
D	Divide:				
	5⟌125				
	25*	15	89	93	93
	All four problems correct	7	68	78	72

*Asterisk indicates correct answer.

sion, which are not generally taught until the third and fourth grades, but that they were more successful with addition and subtraction. Nearly 80% of the 9-year-olds solved the addition problem (which required renaming or carrying), and more than half successfully completed the subtraction problem with renaming (borrowing). Nearly 90% of the 13-year-olds, and over 90% of the 17-year-olds and adults correctly answered the addition, subtraction and division problems; percentages of success were slightly lower on the multiplication problem given.

Thirteen-, 17-year-olds and adults completed computations using integers, fractions, decimals and percents. Two-thirds of the 13-year-olds and over three-fourths of the 17-year-olds successfully added two negative integers. Multiplication of two negative integers proved more difficult: 39% at age 13 and 68% at age 17 answered correctly. The additional 48% of the 13-year-olds and 24% of the 17-year-olds multiplied correctly but gave the wrong sign for their answer.

In computing with unit fractions (fractions with 1 as the numerator), 42% of the 13-year-olds and 66% of the 17-year-olds added two fractions accurately. Results were higher for multiplying two-unit fractions, with 62% at age 13 and 74% at age 17 answering correctly. In working with decimals, 60% at age 13, 78% at age 17 and 74% at adult subtracted 23.8 from 62.1 correctly. Percentages of success for multiplying two decimals were 48% for 13-year-olds and 73% for 17-year-olds. An additional 26% at age 13 and 16% at age 17 multiplied correctly but misplaced the decimal point.

Respondents also used the four operations to solve word problems. A typical 9-year-old problem is presented in Table 2. Forty-six percent of the 9-year-olds determined the necessary operation and multiplied 3 x 7 correctly.

TABLE 2. Exercise and Results: 9-Year-Old Word Problem

An astronaut is to orbit the earth in a space capsule for seven days. If he drinks three pints of water each day, how many pints of drinking water will be needed for the trip?

	Age 9
Respondents answering 21 or 21 pints*	46%

*Asterisk indicates correct answer.

Table 3 presents a typical problem for the three older age levels. In this problem respondents had to divide the speed by the distance traveled to find the answer. Approximately one-third of the 13-year-olds and two-thirds of the 17-year-olds and adults solved this problem correctly.

TABLE 3 Exercise and Results, 13-, 17-Year-Old and Adult Word Problem

If John drives at an average speed of 50 miles an hour, how many hours will it take him to drive 275 miles?

	Age 13	Age 17†	Adult
5½, 5 hrs 30 min, 5½ hrs, 5.5, etc.*	33%	64%	6[illegible]
Wrote down problem right, no or incorrect answer	15	13	8
Answering 5 and 25, 5 hrs. and 25.	11	4	3[illegible]

*Asterisk indicates correct answer.
†Percentages do not total 100% as all response categories are not shown.

Summary

Numbers and numeration include many of the concepts necessary for understanding "basic" mathematics. Lack of ability with these concepts creates a serious handicap in using mathematics in everyday life.

Percentages of success spanned a wide range on these exercises — from close to 100% on exercises such as 3 + 0 and 3 - 0 to under 5% on exercises about ordering fractions and using bases other than 10. People appear to have a good grasp of simple addition and subtraction skills, especially at the 17-year-old and adult levels. Percentages of correct responses were lower on exercises requiring other skills, such as solving word problems or computing with percents.

Tables 4 and 5 display summary statistics for all exercises included in the numbers and numeration content area. Summary statistics cannot give a complete picture of ability in a content area because by their nature they summarize data, removing particular cases and specific exceptions in order to describe the whole in a general fashion. However, the statistics presented do provide a rough gauge of relative abilities for the various age levels.

Table 4 shows the median percentages for all exercises assessed for a particular content area at an age level. The median percentage is the number above and below which half the percentages lie when percentages of success for all the exercises are arranged in rank order. In Table 4, median percentages are first shown for all exercises included in the numbers and numeration content area. Then, median percentages for all exercises in the

TABLE 4. Median Percentages of Success – Numbers and Numeration Exercises*

	Age 9	Age 13	Age 17	Adult
Numbers and numeration (entire chapter) median percentages of success	38%	60%	70%	65%
Number of exercises summarized	(74)	(86)	(74)	(44)
Numeral systems and properties of number systems (Sections 1 and 2) median percentages of success	63	58	65	42
Number of exercises summarized	(36)	(41)	(29)	(16)
Operations with real numbers (Section 4) median percentages of success	30	64	77	75
Number of exercises summarized	(33)	(40)	(41)	(27)

*Exercises on set theory were not summarized, thus, exercise totals for Sections 1, 2 and 4 do not equal the total for the entire chapter.

first two sections in the chapter — Numeral Systems and Properties of Number Systems — and for all exercises in the fourth section — Operations With Real Numbers — are presented so that relative performance in the two areas can be seen. Exercises on set theory were not summarized separately due to the small number of these exercises in the mathematics assessment.

It must be remembered that different exercises were assessed in the different sections of the numbers and numeration content area, and the exercises may not have been of identical difficulty. Thus, the median percentages should be used as an indication of relative performance, not as absolute figures representing a fixed ability level.

Table 5 displays median percentages of success on "overlap" exercises. National Assessment uses the term overlap to designate exercises assessed at more than one age level. Thus, in computing the overlap medians, identical exercise pools are considered for the two age levels being compared. Since the composition of the exercise pool is held constant, comparisons of age-level performance can be made.

Neither the overall medians nor the overlap medians standing alone adequately describe performance. The medians provide a picture of abilities for each age but cross-age comparisons cannot be made as the exercises are not identical. The overlap medians, in contrast, show percentages of success for identical exercises, but the exercises included are not necessarily representative of the overall ability level at each age. The 9-year-old median percentage reflects their ability with all 9-year-old exercises; the overlap medians do not. Readers should consider data from both tables in evaluating performance on the numbers and numeration exercises.

As indicated in Table 4, 9-year-olds displayed greater facility with the numeral systems and properties of number systems than with computation. At the 13- and 17-year-old levels, the advantage was with computational skills. Adults clearly showed greater ability on the computation exercises than on the numeral and number systems exercises.

This does not mean that 13-, 17-year-olds and adults cannot count or write numbers as well as 9-year-olds. Respondents at the upper age levels had greater difficulty with items on representation and ordering of decimals and fractions and in working with different bases, items that generally were not administered at age 9.

TABLE 5. Median Percentages of Success on Overlap Exercises — Numbers and Numeration*

	Age 9 – Age 13		Age 13 – Age 17		Age 17 – Adult	
Numbers and numeration – median percentages of success	29%	80%	51%	72%	69%	63%
Number of exercises summarized	(43)	(43)	(67)	(67)	(43)	(43)

*Numbers of overlap exercises were not sufficient for separate analysis of numeral systems, properties of number systems or operations with real numbers.

CHAPTER 2

MEASUREMENT

We use measurement to quantify many things in our world, from the simplest to the most complex. The child using a ruler is determining the number of units in a given distance; the scientist calculating the distance to the moon is doing the same thing, although the units are defined differently and the distance cannot be "measured" directly.

The measurement exercises that formed part of the mathematics assessment included recognition of various types of measurement units and their relationships, use of measurement instruments, conversion of quantities from one unit to another and application of metric geometry. The exercises concerned measures of time, temperature, weight, length, capacity and area.

Representative 9-Year-Old Exercises

Two 9-year-old exercises concerned measurement of time, one involving calendar time; the other, clock time. Forty-five percent of the 9-year-olds correctly answered that the date one week after July 4 is July 11. One-fourth of the 9-year-olds knew that the amount of time between 4:25 and 5:00 was 35 minutes.

Nine-year-olds were also asked to tell time when shown a model of a clock. Ninety-six percent of the 9-year-olds accurately gave the time when the clock was set on the hour. Approximately 80% were successful when the time shown was half past the hour, and 74% were able to set the clock to indicate 15 minutes past the hour.

In working with several other units of measurement, slightly over two-thirds of the 9-year-olds determined the amount of money equaled by one quarter, two nickels and four pennies; 44% of the 9-year-olds and 84% of the 13-year-olds correctly gave the number of quarts in a gallon.

Use of Measurement Instruments

Nine-year-olds and 13-year-olds were asked to read two measuring instruments — a thermometer and a ruler. On both exercises, percentages of success varied according to the size of the gradations involved. When shown a model of a thermometer indicating a temperature that was a multiple of 10, 92% at age 9 and 96% at age 13 read the temperature correctly. However, results were lower — 19% for age 9 and 55% for age 13 — when respondents had to use gradations of two degrees.

The size of the intervals used also affected performance on reading a ruler. Nine-year-old and 13-year-old respondents were shown four marked points on a ruler. Naming the four points required the ability to read whole inches, half inches, quarter inches and eighth inches. Percentages of success on each of these categories are shown in Table 6. Respondents at ages 9 and 13 also measured the width and length of a board. The task proved easier when the distance to be measured was under 12 inches; the difficulty was greater when the distance was over 12 inches and the ruler had to be moved. Results for this exercise are also shown in Table 6.

Respondents at all four age levels used a ruler to determine the thickness of the bottom of a box having different internal and external dimensions. The difference was in units of

TABLE 6. Percent Correctly Reading Various Interval Marks on a Ruler

	Age 9	Age 13
Read whole inches	84%	93%
Read half inches	60	83
Read quarter inches	14	54
Read eighth inches	1	25
Measured width of board (whole inches under 12)	82	91
Measured length of board (whole inches over 12)	47	73

whole inches. Eighteen percent at age 9, 43% at age 13, 60% at age 17 and 64% at adult found the correct difference.

Comparison and Conversion of Measurement Units

One exercise, administered at all four age levels, examined the ability to compare measurement units. Respondents were asked to determine which of two amounts, given in different units, was larger for five types of measurement. The exercise and results are shown in Table 7. The relationship between feet and yards was most obvious for respondents at the three upper age levels. Nine-, 13- and 17-year-olds had the most difficulty with the relationship of pounds and ounces.

TABLE 7 Exercises and Results Measurement Comparisons

	Age 9	Age 13	Age 17	Adult
Feet–yards	82%	94%	97%	96%
Pints–quarts	83	92	94	95
Nickels–dimes	83	92	93	89
Weeks–months	54	87	94	99
Ounces–pounds	36	73	85	92

Several items dealing with conversion of units were administered at ages 13, 17 and adult. Units of time, weight, length and capacity were included. Results for these exercises, which included units of time, weight, length and capacity, are shown in Table 8. As the table indicates, the ability to convert units, with the exception of the months and years exercise, was quite similar for different types of measures.

TABLE 8. Results for Five Exercises About Conversion of Units: Ages 13, 17 and Adult

	Age 13	Age 17	Adult
30 months = x years, x months	76%	92%	92%
Number of ounces in given number of pounds	43	64	73
Number of yards in given number of feet	47	65	66
Number of pints in given number of gallons	44	60	68
6'7" closer to 84" than to 66, 72 or 90 inches	46	66	75

Metric Geometry

Respondents also used geometric formulas to calculate measurements. Seven percent of the 13-year-olds and slightly over 25% of the 17-year-olds and adults correctly found the area of a square with a perimeter of 12 inches. Seventeen-year-olds and adults were more successful in using a geometric formula to find the number of gallon cans of paint needed to cover a 48-foot x 10-foot area when one can covered 250 square feet. Two-fifths of the 17-year-olds and three-fifths of the adults correctly stated that two cans of paint would be needed.

Summary

Respondents seemed fairly successful in applying measurement concepts. Median percentages of success and overlap medians are shown in Table 9. As discussed in Chapter 1, overall medians for all the measurement exercises should not be compared across ages since the exercises given to each age level were not identical. Overlap medians do provide a means for age-level comparisons but are not a complete picture of each age level's performance. The two statistics should be considered together.

The median percentage of 65% for 17-year-olds indicates that on half of the 29 exercises administered to 17-year-olds, over 65% of the respondents answered correctly. For adults, on half of the 29 exercises administered, over 73% responded correctly. The overall medians and overlap medians for 17-year-olds and adults are the same because the 17-year-old and adult items in this content area were identical.

Nine-year-olds were most successful with simple manipulations of measuring instruments. Over 90% could read a thermometer set at a multiple of 10 degrees and set a clock to an even hour; 84% could use a ruler to make a measurement in whole inches. Percentages of correct responses were also high for comparisons of pints and quarts, nickels and dimes and feet and yards. The 9-year-olds had considerable difficulty in using a ruler to measure quarter and eighth inches.

Patterns of success at age 13 were similar to those for 9-year-olds. Over 90% correctly compared quantities of pints and quarts, nickels and dimes and feet and yards. Ninety-three percent successfully used a ruler to measure in whole inches, and 83% correctly measured half inches. Thirteen-year-olds also found measuring to eighth inches troublesome.

On the majority of the measurement exercises, adults held an advantage over 17-year-olds. Adult performance was furthest above that of 17-year-olds — 20 percentage points — on the problem about the number of cans of paint needed to cover a certain area. Adults were also generally more successful in making conversions from one type of unit to another.

TABLE 9. Median and Overlap Median Percentages of Success — Measurement Exercises

	Age 9	Age 13	Age 17	Adult
Median percentages of success	46%	63%	65%	73%
Number of exercises summarized	(35)	(35)	(29)	(29)

	Age 9 – Age 13		Age 13 – Age 17		Age 17 – Adult	
Overlap median percentages of success	45%	73%	59%	78%	65%	73%
Number of exercises summarized	(22)	(22)	(19)	(19)	(29)	(29)

CHAPTER 3

GEOMETRY

Geometry in [illegible] primary grades consists mainly of learning the names of various shapes and figures and understanding the concept of two- and three-dimensional space. Students in the upper elementary grades apply formulas to calculate perimeter, area and volume. They also use instruments such as the straightedge, compass and protractor to construct and measure geometric figures. Secondary school students may or may not study geometry further depending on the mathematics courses that they take. The National Assessment of Educational Progress (NAEP) mathematics assessment was concerned mainly with the informal geometry encountered in the elementary grades prior to formal geometry courses. Thus, no geometric proofs were included and few exercises required skills from specific geometry courses.

Identification of Geometric Figures

Respondents at all four age levels were asked to supply the correct geometric names when shown actual models of eight geometric figures. Percentages of correct responses for the various objects are given in Table 10. At all ages, percentages of success were highest for naming the circle.

Nine-year-olds were also asked to identify drawings of a rectangle and a triangle. About three-fourths of the 9-year-olds identified a drawing of a rectangle when alternative choices included a parallelogram, a trapezoid and a triangle. Slightly less (72%) of the 9-year-olds properly named a triangle.

Almost half of the 9-year-olds were able to identify parallel lines when shown several different configurations of lines. However, only 18% at age 9 successfully drew a line parallel to a given line.

TABLE 10. Percentages of Correct Responses to Question "What is the Name of This Figure?"

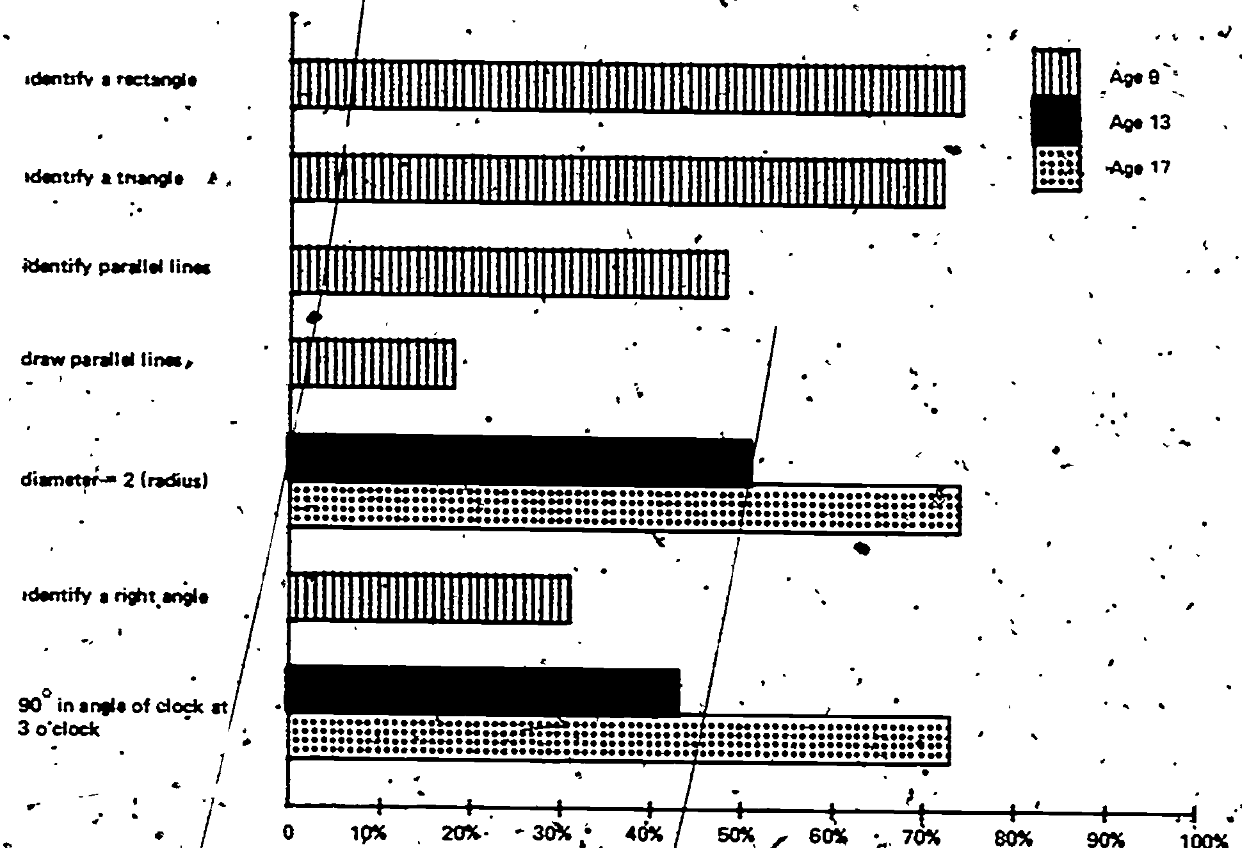
	Age 9	Age 13	Age 17	Adult
Circle	96%	95%	97%	91%
Triangle	88	89	92	89
Cone	28	54	74	72
Cylinder	3	24	53	56
Cube	4	23	41	52
Sphere	2	21	46	41
Ellipse	2	3	11	10

Approximately half of the 13-year-olds and three-fourths of the 17-year-olds demonstrated that they knew the relationship between the radius and the diameter of a circle ($d = 2r$).

About one-third of the 9-year-olds were able to identify a right angle when given the alternatives right, acute and obtuse. Forty-three percent of the 13-year-olds and 73% of the 17-year-olds stated correctly that there are 90° in the angle formed by the hands of a clock at 3 o'clock.

Figure 5 compares results on the geometric identification exercises, excluding the exercise shown in Table 10.

FIGURE 5. Results for Selected Exercises: Geometry – Identification of Geometric Figures



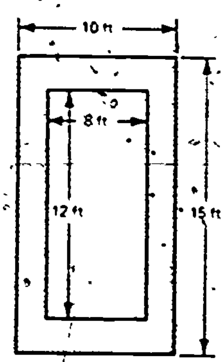
Geometric Calculations

Nine-year-olds had difficulty with the concept of perimeter in a word problem. Only 7% figured out how much fence would be needed to enclose a 9-foot x 5-foot garden. Forty-three percent of the 9-year-olds added 9 and 5 for an answer of 14, and 16% responded with "I don't know."

Seventeen-year-olds and adults were asked to find the area of the shaded portion of the figure opposite.

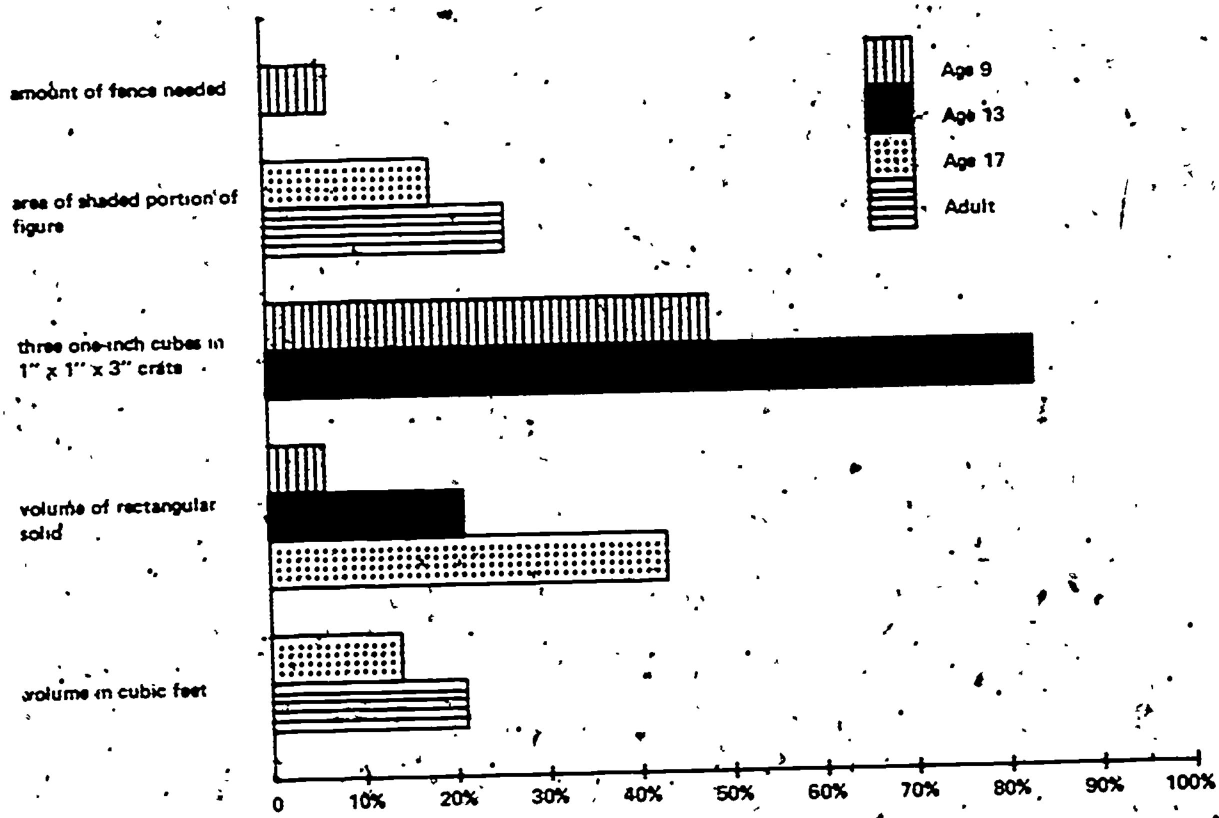
Eighteen percent of the 17-year-olds and 26% of the adults did so successfully.

Several exercises required calculation of volume. When shown pictures of a one-inch cube and a crate 1" x 1" x 3", 48% of the 9-year-olds and 83% of the 13-year-olds correctly



responded that it would take three cubes to fill the crate. Nine-, 13- and 17-year-olds were shown a picture of a rectangular solid marked off in cubes and asked to determine the total number of cubes it contained. Six percent of the 9-year-olds, 21% of the 13-year-olds and 43% of the 17-year-olds did so successfully. Fourteen percent at age 17 and 21% at adult correctly calculated a volume in cubic feet

FIGURE 6. Results for Selected Exercises: Geometric Calculations



when two dimensions were given in feet and one in inches. Figure 6 displays results for the geometric-calculation exercises.

Use of a Protractor

Thirteen-year-olds, 17-year-olds and adults used a protractor to measure two acute angles and one obtuse angle. On the two acute angles, percentages of success were 33% and 36% for 13-year-olds, 57% and 60% for 17-year-olds and 47% and 48% for adults. Percentages of success on the obtuse angle were the following: 31% at age 13, 52% at age 17 and 44% at adult.

Construction

Thirteen-year-old and 17-year-old respondents were asked to perform a geometric construction — bisecting an angle using a straightedge and compass. Ten percent of the 13-year-olds and 37% of the 17-year-olds made an adequate construction; 55% of the 13-year-olds and 30% of the 17-year-olds responded with "I don't know."

Summary

Identification of familiar geometric figures seemed the easiest part of the geometry assessment for respondents. Of course, some figures were a good deal more familiar than others. The circle and the triangle were most often named correctly by all age levels, while the cube, the sphere and the ellipse proved more difficult to identify at all ages.

Respondents were generally more successful at recognizing the name of a figure in a list of multiple-choice alternatives than in recalling the name when shown a model of the figure. For example, 21% at age 9, 66% at age 13,

82% at age 17 and 81% at adult correctly selected cylinder from a list of alternatives when asked the question, "Which geometric figure is shaped most like a (familiar object)?" In contrast, 3% of the 9-year-olds, 24% of the 13-year-olds, 53% of the 17-year-olds and 56% of the adults named a cylinder when shown a plastic model of the figure.

Some of the metric geometry exercises proved difficult for respondents. The majority of 9-year-olds were not able to solve the area and volume problems included in the assessment: under 50% at age 9 got any of these problems right. Thirteen-year-olds also had difficulty in calculating volume. Although 83% knew that three one-inch cubes would fit into a crate 1" x 1" x 3", when shown pictures of the respective sizes of the figures, only 21% correctly determined the volume of a rectangular solid marked in cubes. Forty-three percent at age 17 found the volume of the rectangular solid; however, less than one-fifth at age 17 and 21 to 26% of the adults found the area of the shaded border shown on page 16 and correctly figured a volume in cubic feet.

As in previous chapters, median percentages of success and overlap percentages of success are presented (see Table 11) to give an idea of overall performance in the geometry content area at each age level and to allow some comparison of relative performance of the age levels. As explained in Chapter 1, extreme care must be used in interpreting these numbers. Both the overall and the overlap medians should be considered in evaluating performance. It also must be remembered that these numbers represent success on the National Assessment geometry exercises, not success on the entire subject-matter area defined as "geometry."

TABLE 11. Median and Overlap Median Percentages of Success — Geometry Exercises

	Age 9	Age 13	Age 17	Adult
Median percentages of success	28%	51%	57%	53%
Number of exercises summarized	(39)	(37)	(37)	(29)

	Age 9 — Age 13		Age 13 — Age 17		Age 17 — Adult	
Overlap median percentages of success	28%	58%	47%	70%	60%	53%
Number of exercises summarized	(27)	(27)	(29)	(29)	(29)	(29)

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CHAPTER 4

VARIABLES AND RELATIONSHIPS

The study of variables and relationships provides a foundation for the study of higher mathematics. Variables are necessary to talk about general rather than particular cases. The relationships between variables are important in developing logical mathematical systems. Topics included in the area of variables and relationships are algebraic expressions, equations and inequalities, relations, functions and graphs, exponents and trigonometry and mathematical logic. The problems are generally of the type that would be found in prealgebra or elementary algebra courses. The National Assessment of Educational Progress (NAEP) did not assess advanced mathematical subjects — for example, matrix algebra, vector algebra, or calculus — due to the relatively small number of people exposed to such subjects in the schools.

Although some ideas involving variables and their relationships are taught in the early elementary grades, variables and their relationships are usually not studied in depth until students enter prealgebra and algebra courses. These courses are generally first available at the seventh- and eighth-grade levels. Thus, the assessment exercises concentrated upon 13-year-old, 17-year-old and adult skills in using variables and relationships.

A number of exercises were administered only to 17-year-olds. Thirteen-year-olds were not expected to have learned the required skills, and it was anticipated that adults might become discouraged if confronted by problems that they had forgotten how to solve. The exercises given only to 17-year-olds were designed to discover what students knew and could do near the conclusion of their high school career.

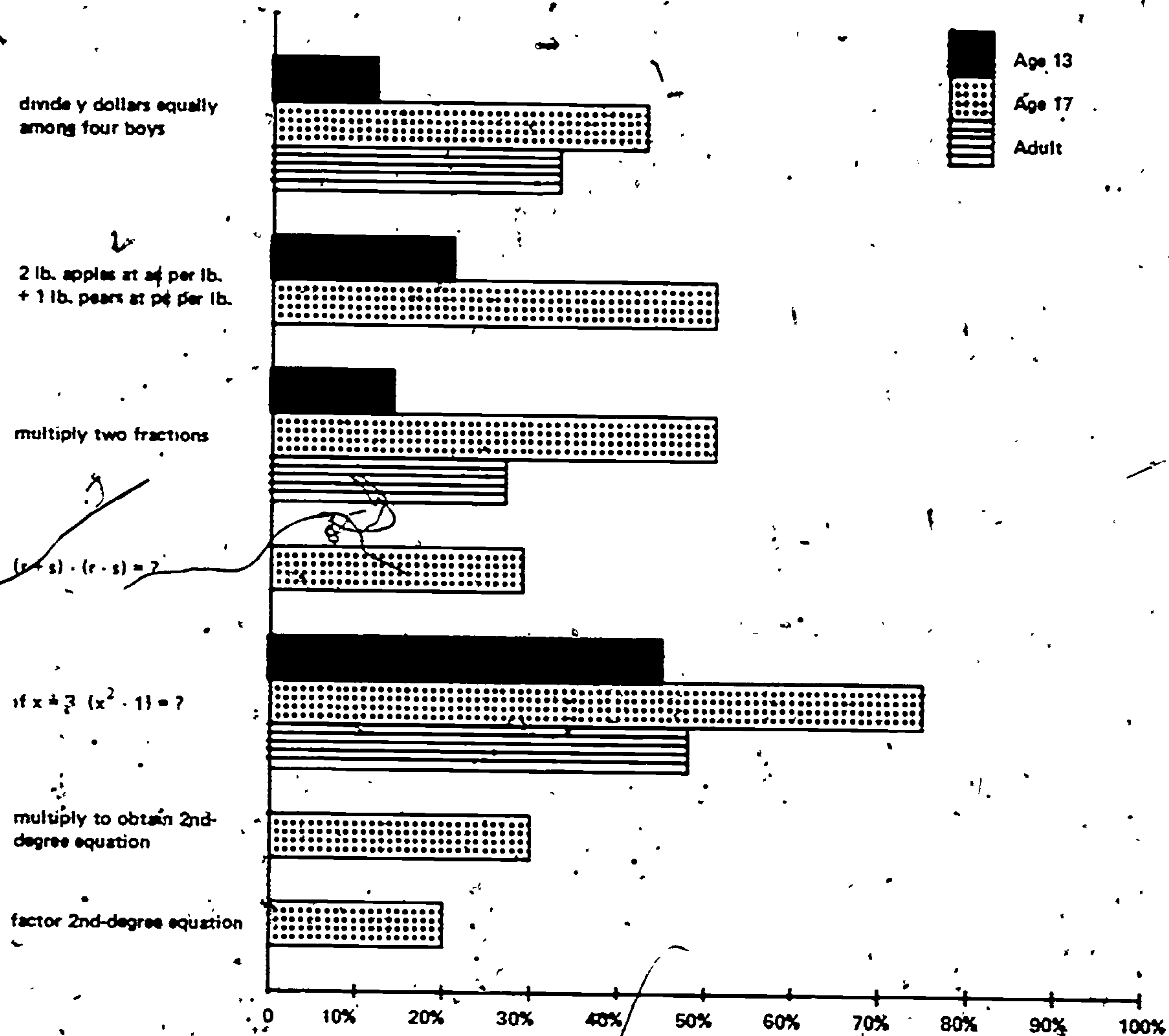
Algebraic Expressions

In studying algebra, students must in effect learn an entirely new language. They discover that a variable serves as a replacement for any element of a given set. They can then manipulate these variables and perform arithmetic operations.

In answer to the question "If y dollars are shared equally among four boys, how many dollars does each boy receive?" 12% of the 13-year-olds, 43% of the 17-year-olds and 33% of the adults correctly responded with $4/y$, or an equivalent form. Approximately one-fifth of the 13-year-olds and half of the 17-year-olds correctly gave the algebraic expression for the cost of two pounds of apples at a¢ per pound and one pound of pears at p¢ per pound ($2a + p$, $a + a + p$, etc.). Fourteen percent of the 13-year-olds, 51% of the 17-year-olds and 27% of the adults successfully multiplied two fractions, one of which included a variable. At age 17, 29% apparently understood the relation of the parenthesis and the negative sign, successfully manipulating the following expression — $(r + s) - (r - s)$ — to find the answer $2s$ or $s + s$.

One exercise asked respondents to substitute a value for a variable to find a value of a second-degree equation (if $x = 3$, $x^2 - 1 = ?$). Percentages of success for 13- and 17-year-olds and adults were 45%, 75% and 48%, respectively. Two exercises dealing with higher-order equations, one requiring multiplication, the other demanding factoring, were administered only to 17-year-olds. Thirty percent completed the multiplication successfully; 20% gave the correct factors. Figure 7 shows percentages of success on the exercises about using algebraic expressions.

FIGURE 7. Results for Selected Exercises: Algebraic Expressions



Equations and Inequalities

The complexity of equations and inequalities ranges from filling in the missing number in a simple open sentence to factoring quadratic equations and beyond. As with many of the other topics in the mathematics assessment, the concepts of equations and inequalities are introduced in the elementary grades and amplified in later mathematics courses.

Nine-year-olds were asked to give the number that makes the following open sentence true: $3 + \square = 10$. Ninety percent were able to do so. A similar equation, $x - 3 = 7$, was solved by 49% at age 9, 85% at age 13, 94% at age 17 and 81% at adult. Seventeen-year-olds were considerably more successful than 13-year-olds in solving the equation $3x - 3 = 12$, with 75% of the 17-year-olds and 39% of the 13-year-olds correctly answering 5.

A more complicated equation, requiring use of the additive inverse to group quantities on either side of the equal sign, was solved correctly by [illegible]% of the 17-year-olds and 25% of the adults. Half of the 13-year-olds, 70% of the 17-year-olds and over half the adults (56%) correctly determined that if $x < 4$, $x + 7$ must be less than 11.

Thirteen- and 17-year-olds were asked to convert Fahrenheit temperatures to Centigrade given the following information: using the formula $F = 9/5\,C + 32°$, what is C when $F = 77°$? Respondents not only had to substitute into the formula but had to use the additive and multiplicative inverses to solve the equation. There was a substantial difference in skill on this exercise between ages 13 and 17. Two percent of the 13-year-olds and 24% of the 17-year-olds gave the correct answer (25° C), a 22-percentage-point difference. Percentages of success for each age level on the exercises described in this section appear in Figure 8.

Relations, Functions and Graphs

Seventeen-year-olds showed their understanding of functional notation in answering the question, "If $f(x) = x + 1$, $f(2) = ?$" Two out of five 17-year-olds correctly answered either 3 or 2 + 1.

A number of exercises concerned graphing of equations. Exhibit 2 shows an exercise asking respondents to identify the graph of an equation. Under 15% of the 13-year-olds and less than half of the 17-year-olds identified the proper graph. About one in five of the 17-year-olds was able to write the equation

FIGURE 8. Results for Selected Exercises: Equations and Inequalities

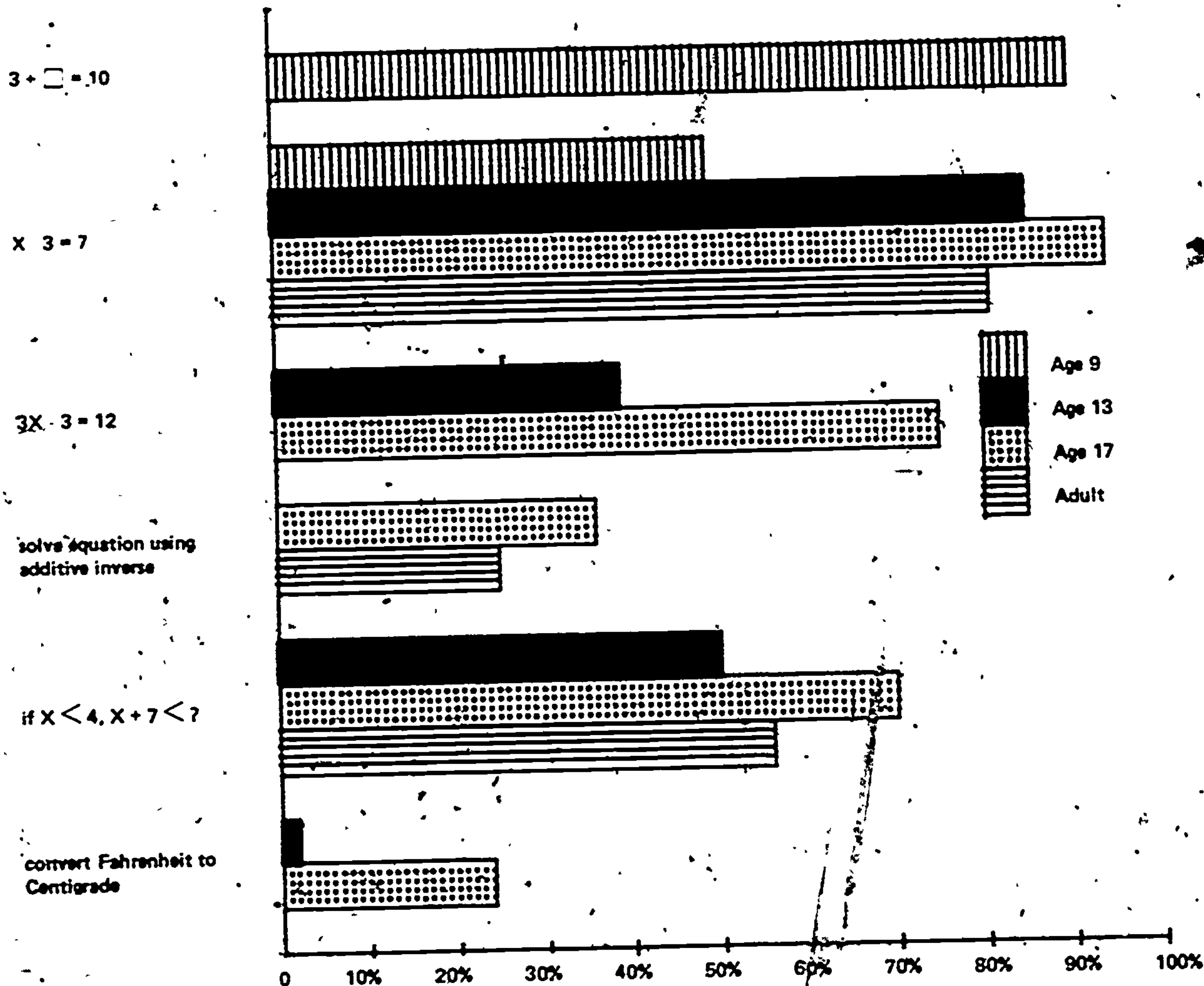
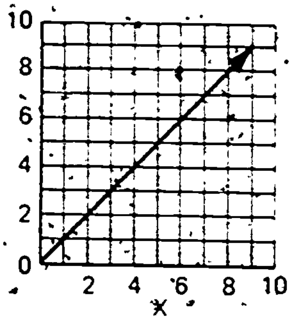
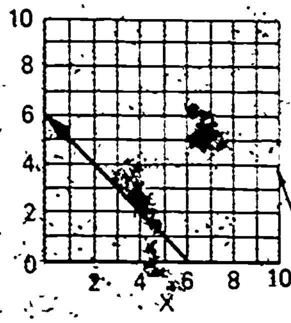
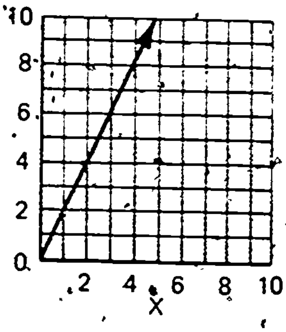
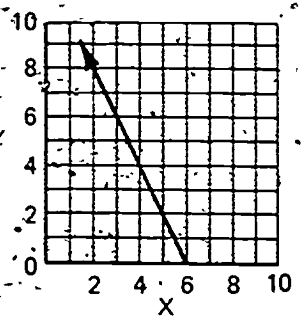


EXHIBIT 2. Exercise and Results: Graph of an Equation

Which chart shows part of the graph of the equation $x = y$?

Age 13	Age 17		Age 13	Age 17	
13%	46%		74%	43%	
2	2		2	2	
8	6	I don't know.			
0	1	No response			

for a horizontal line with a y-intercept of 2, and one in five was able to draw a graph of a given equation. Seventeen-year-olds were also asked to give the slope and y-intercept of a particular equation when no graph was shown. Sixteen percent successfully determined the slope and 12% gave the proper y-intercept.

Special Types of Functions: Exponents and Trigonometry

Exponents and trigonometric functions are special classes of functions having properties that set them apart from other functions. Exponential expressions are represented graphically by curves rather than lines. The trigonometric functions are the ratios of certain sides of right triangles to specified other sides.

The more sophisticated exponential and trigonometric functions are generally taught in intermediate algebra, which is not studied by a majority of high school students. Results should be evaluated in light of that fact.

Several National Assessment exercises dealt with the meaning of exponential expressions. Fifty percent of the 13-year-olds and 74% of the 17-year-olds correctly stated that $4^3 = 64$

or 4 x 4 x 4. Percentages were lower for less-common expressions such as a number to the zero-power (17% at age 13 and 28% at age 17), a number to a fractional power (19% at age 17) and a number to a negative power (20% at age 17). Respondents were also asked to give a value for a number expressed in scientific notation. Thirty-seven percent of the 13-year-olds, 62% of the 17-year-olds and 49% of the adults correctly chose 360 as the value of 3.6×10^2.

Other exercises measured the ability to understand radical signs and take square roots. At ages 13, 17 and adult, 37%, 75% and 60%, respectively, were able to give the square root of 16. Respondents had more difficulty in finding a square root that was not perfect. Half of the 17-year-olds and slightly more than one-third of the adults correctly determined that 3.2 was closer to the square root of 10 than 2.5, 2.7 or 3.8. Seventeen-year-olds and adults were also asked to use a table of square roots. Seventeen percent of the 17-year-olds and 18% of the adults located the square root of a two-digit number. A smaller number, 8% at age 17 and 12% at adult, found the square root of the same number expressed in hundreds (two zeros added to the two-digit number).

Questions about trigonometric functions were asked only of 17-year-olds. Around 11% of the 17-year-olds were able to identify each of three common trigonometric functions. Percentages of success on exercises concerning exponents and trigonometry are found in Figure 9.

Mathematical Logic

Mathematics is a logical system with each new part of the system building upon what went before. Formal logic is concerned with the truth or falsity of statements and the effect that the form of a statement has upon its truth or falsity. Informal logic, the type that most people are familiar with, involves the use of an orderly process of reasoning to arrive at a problem's solution. National Assessment measured respondents' facility with informal logic, considering mainly their ability to determine relationships between various statements.

A typical example of a logic exercise and results for the exercise appear in Table 12. Half the 17-year-olds and slightly over half the adults determined the proper relationship. One need not know any specific "rules" of logic to solve this problem, but one must rigorously eliminate any alternatives where there is not enough information to determine the absolute truth or falsity of the statement.

A logic problem administered only to 13-year-olds concerned the relative ages of children. The problem was the following: "John is 4 years older than Ellen and Ellen is 11 years younger than Monica. Monica is 12 years old. How old is John?" Seventy-one percent of the 13-year-olds answered correctly that John was 5. A similar unreleased exercise about relative ages using inequalities (greater than, less than) rather than actual numbers was answered correctly by 54% of the 9-year-olds, 81% of

TABLE 12. Percentages of Success for Typical Logic Exercise

Which one of the statements below follows logically from the statement, "All good drivers are alert"?

	Age 17	Adult
◯ All alert persons are good drivers.	24%	26%
◯ Some alert persons are not good drivers.	10	7
◯ A person who is not a good driver is not alert.	14	9
⬤ A person who is not alert is not a good driver.	50	56
◯ I don't know.	2	1
No response	+*	+*†

*Plus equals rounded percents less than one.
†Figures may not add to 100% due to rounding error.

FIGURE 9. Results for Selected Exercises: Exponential and Trigonometric Functions

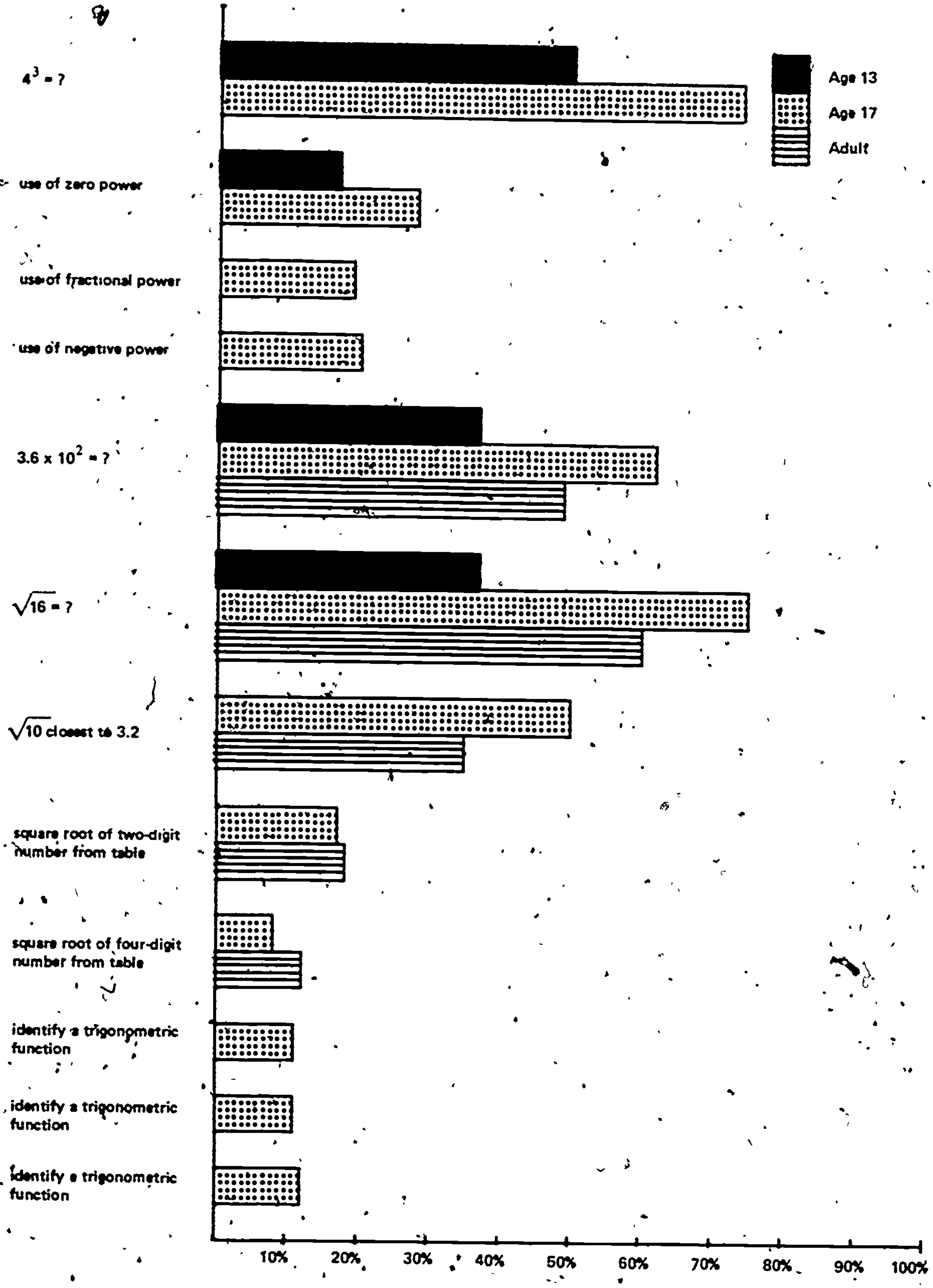
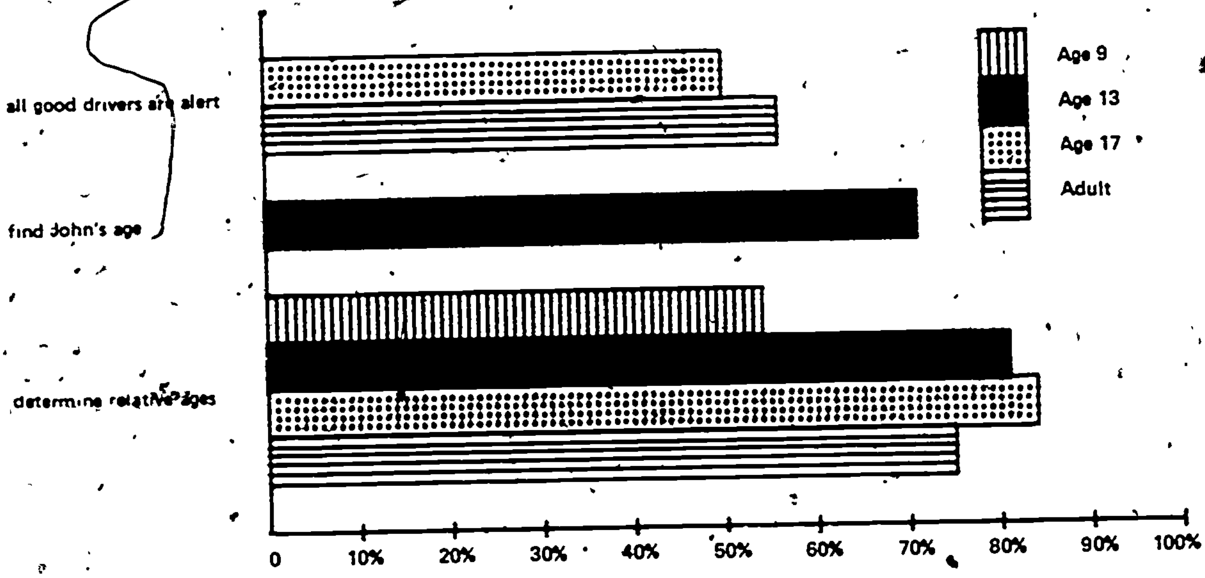


FIGURE 10. Results for Selected Exercises: Logic



the 13-year-olds, 84% of the 17-year-olds and 75% of the adults. Percentages of success on logic exercises are presented in Figure 10.

Summary

At all three age levels, respondents did best on the logic exercises and on solving simple equations and inequalities. Thirteen-year-olds had more difficulty in solving equations including multiplication than those simply using addition and subtraction. Thirteen-year-olds did not seem particularly familiar with manipulation of variables. Twenty-one percent successfully stated the price of two pounds of apples at a¢ per pound and one pound of pears at p¢ per pound in symbolic form; 14% were able to multiply two fractions, one of which had a variable in the numerator, and 12% correctly expressed y dollars divided equally among four boys.

Adult performance followed a pattern similar to that of the 13-year-olds, with the highest percentages of success on solving simple equations and inequalities and problems in logic. Adults showed greater ability than 13-year-olds on two exercises about manipulating symbols; 27% of the adults successfully multiplied the two fractions, and 33% correctly expressed y dollars divided equally among four boys.

Seventeen-year-olds experienced the most difficulty with exercises that were not assessed at the other two age levels — exercises on quadratic equations, zero, negative and fractional exponents, logarithmic functions and graphing equations. Their performance on the other exercises showed the same pattern as at the other ages although percentages of success were generally higher.

To provide a generalized summary of results for variables and their relationships, overall medians and overlap medians are presented in Table 13. These summary statistics are subject to the limitations discussed in Chapter 1, page 9. The overall medians give a feel for how well an age level did on all exercises; the overlap medians provide a comparison of age performance of different age levels on identical exercises.

As is evident from Table 13, nearly all of the 13-year-old and adult exercises were also administered at age 17. Seventeen-year-olds had a distinct advantage on these "overlap" exercises; the overall median for 17-year-olds was lowered by those exercises administered only to 17-year-olds.

TABLE 13. Median and Overlap Median Percentages of Success – Variables and Relationships Exercises

	Age 13	Age 17	Adult
Median percentages of success	39%	38%	49%
Number of exercises summarized	(28)	(50)	(21)

	Age 13 – Age 17		Age 17 – Adult	
Overlap median percentages of success	39%	67%	64%	49%
Number of exercises summarized	(24)	(24)	(21)	(21)

40

CHAPTER 5

PROBABILITY AND STATISTICS

What are the chances of rolling a seven? Of drawing a fourth queen? Any good gambler knows that he has to use probability if he is to win more often than he loses, although he may not call it by that name. Businessmen and government officials use probability and statistics to "gamble" on a different level — for example, to predict next year's sales or to estimate where government allocations will be needed.

Although probability and statistics are closely allied, they are not one and the same thing. Probability concerns the chances of an uncertain event happening. Statistics is often separated into two categories, descriptive and inferential statistics. Descriptive statistics includes methods of describing large quantities of data; inferential statistics uses concepts of probability to draw conclusions about a large group from a smaller subset or sample of that group.

Probability of Events

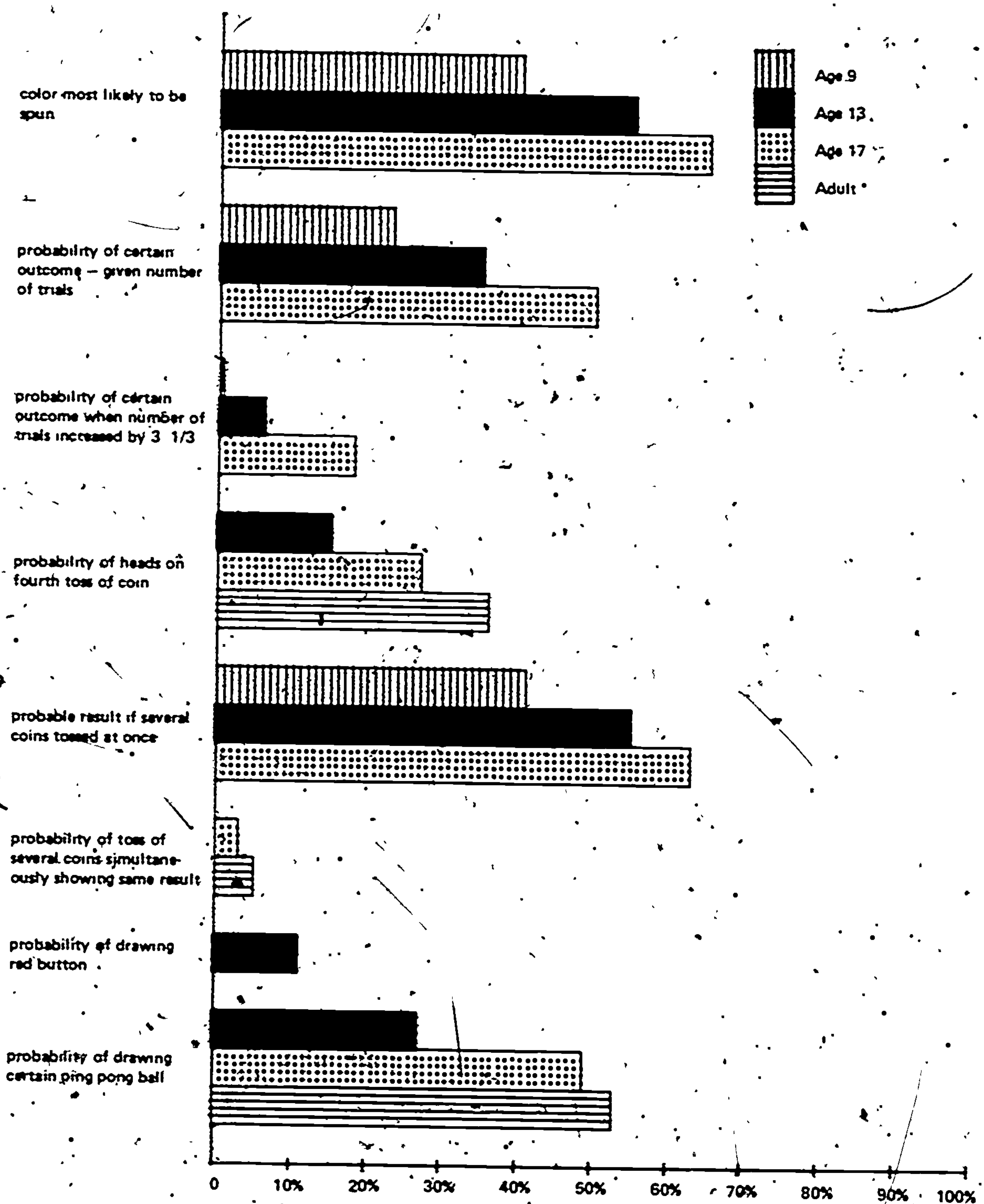
Students often become acquainted with probability through simple experiments such as flipping a coin or spinning a spinner. After they have empirically determined the ratio of the occurrence of a given event to the total number of chances, they discover the general principles for determining probability.

When shown a spinner with several different colored sections, 40% at age 9, 55% at age 13 and 65% at age 17 correctly stated that the color that occurred the most often on the spinner would be most likely to be spun. Respondents seemed to have some difficulty in recognizing that the number of attempts does not affect the probability of a particular occurrence. When asked to determine the probability of a certain outcome for a fairly low number of spins of a spinner, 23% of the 9-year-olds, 35% of the 13-year-olds and 50% of the 17-year-olds gave the correct answer; however, only 1% of the 9-year-olds, 6% of the 13-year-olds and 18% of the 17-year-olds gave the correct answer when the number of attempts was increased three and one-third times.

Several assessment exercises involved the probabilities of coin flipping. Of course, coin flipping is an excellent example of probability because for each event there are only two possible outcomes, heads or tails, and only one can occur. Fifteen percent of the 13-year-olds, 27% of the 17-year-olds and 36% of the adults stated that there is a 50-50 chance of getting "heads" on the fourth toss of a coin if three previous tosses have resulted in two "heads" and one "tails." Asked the probable result if several coins were tossed at the same time, 41% at age 9, 55% at age 13 and 63% at age 17 correctly determined that half the coins should show heads and half tails. A more difficult problem involved the chances of all the coins tossed showing the same results. Only 3% of the 17-year-olds and 5% of the adults answered this problem correctly.[4]

Two exercises concerned the possibility of selecting a particular object from a varied assortment. Approximately 1 in 10 (11%) at age 13 correctly stated that there was a 1 in 6 chance of drawing a red button first from an assortment of one red and five black buttons.

FIGURE 11. Results for Selected Exercises: Probability of Events



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Twenty-seven percent at age 13, 49% at age 17 and 53% at adult correctly determined the chances of drawing a particular ping pong ball from an assortment.

Figure 11 shows results for the exercises discussed in this section. Percentages of success for these exercises were not high; no more than two-thirds of the respondents responded correctly on any of these exercises and in most cases the percentage of correct responses was considerably lower.

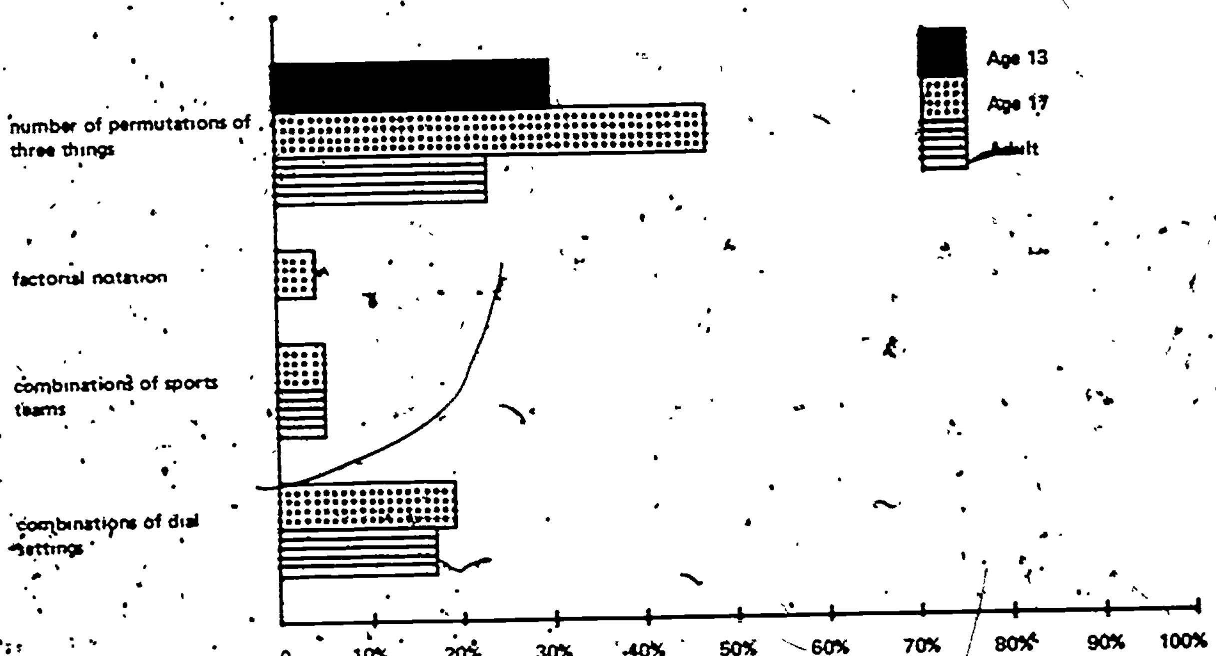
Permutations and Combinations

A permutation is the number of different arrangements that can be made of a certain number of elements (either some or all) of a set. A combination, on the other hand, is not concerned with the arrangement or order of the elements but merely with the number of different combinations that can be made from a specific number of elements of a given set. For example, the number of ways three people can arrange themselves in going through a line is a permutation. The number of games five basketball teams have to play in order that each team plays all the other teams once is a combination.

In answer to the question about the possible permutations of three people going through a line, 30% of the 13-year-olds, 47% of the 17-year-olds and 23% of the adults correctly answered that there were six arrangements. An exercise administered only to 17-year-olds concerned factorials, which are used to solve permutation and combination problems. A small percentage of the 17-year-olds, 4%, gave the proper value for a number written in factorial notation (e.g., 3! = ?).

A combination problem similar to the example about the basketball teams was administered to 17-year-olds and adults. Five percent at both age levels got the problem right. Twenty-four percent of the 17-year-olds and 20% of the adults evidently squared the number of elements to reach their answer; 9% at age 17 and 15% at adult divided the number of elements in half. Another question concerned the number of combinations possible on a combination lock with several dials; 19% of the 17-year-olds and 17% of the adults answered successfully. Eighteen percent at age

FIGURE 12. Results for Selected Exercises: Permutations and Combinations



17 and 29% at adult responded with "I don't know." Figure 12 displays results for these exercises. Similar to results for the previous section, percentages of success are not high.

Statistics

Exercises in this section involved definition of some elementary statistical terms. The statistics taught in elementary and secondary schools are primarily descriptive statistics; inferential statistics are not ordinarily introduced until the college level.

Thirty-eight percent of the 13-year-olds, 66% of the 17-year-olds and 69% of the adults properly computed a simple average. Nineteen percent at age 17 and 38% at adult solved a problem using a weighted average. Median did not seem to be as familiar a term as average; 12% at age 13, 15% at age 17 and 18% at adult correctly identified the median of a series of numbers.

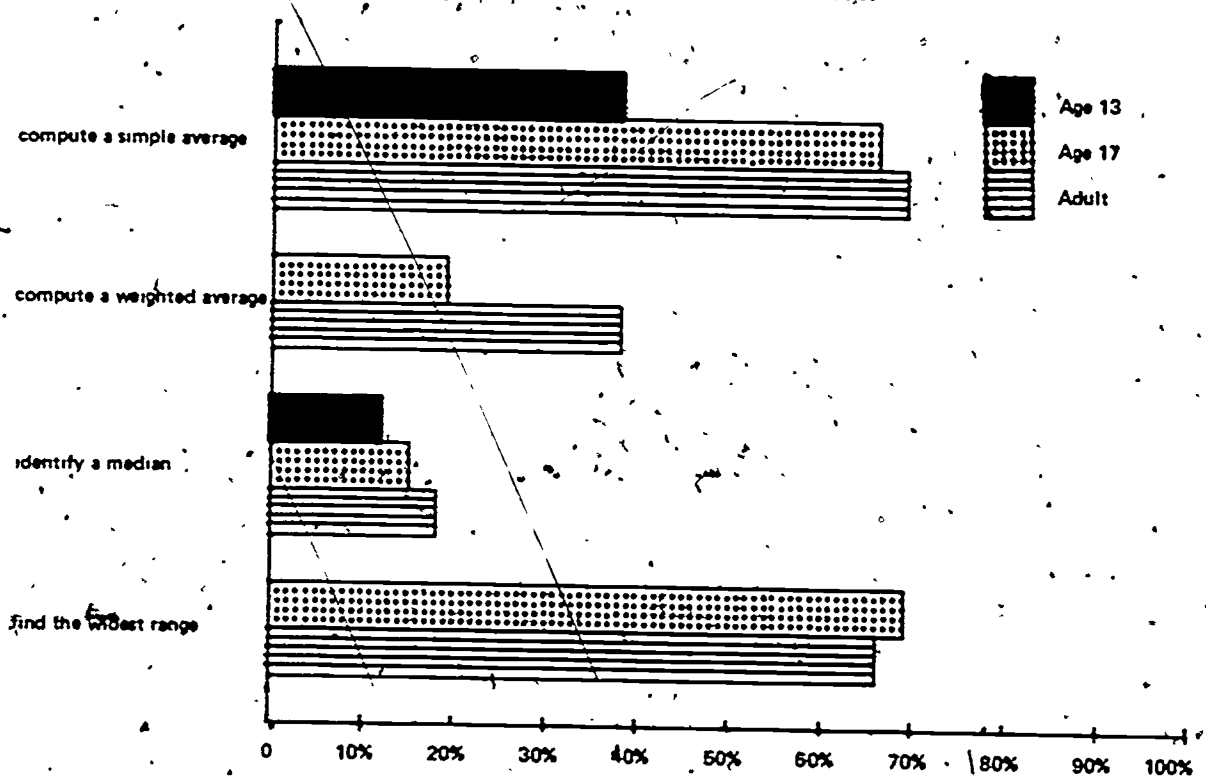
In another definitional exercise, 69% of the 17-year-olds and 66% of the adults correctly selected the group showing the widest range for a given characteristic. Percentages of success for these exercises appear in Figure 13.

Summary

Respondents did not appear extremely familiar with concepts of probability and statistics. Results were under 70% for all exercises summarized in this section.

Thirteen-year-olds had their greatest percentage of correct responses, 55%, on two probability exercises: choosing the color most likely to be spun and determining how many coins would probably come up the same if a number of coins were tossed at the same time. Seventeen-year-olds and adults did best on the question about the widest range for a given characteristic and on determining a simple average.

FIGURE 13. Results for Selected Exercises: Statistics



On most of the probability and statistics exercises, adults had a slight advantage over 17-year-olds. The adult advantage tended to be on exercises about chances or probabilities. Adult performance was close to or below that of 17-year-olds on exercises about permutations and combinations.

Table 14 presents median percentages and overlap median percentages for 17-year-olds and adults. The number of 13-year-old exercises was too small to allow meaningful summaries.

TABLE 14. Median and Overlap Median Percentages of Success – Probability and Statistics Exercises

	Age 17	Adult
Median percentages of success	27%	32%
Number of exercises summarized	(17)	(12)

	Age 17 – Adult	
Overlap median percentages of success	29%	32%
Number of exercises summarized	(12)	(12)

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CHAPTER 6

CONSUMER MATHEMATICS

In times such as the present, when income never seems to keep up with the rising rate of inflation, the consumer more than ever needs mathematical skills. Comparing the advantages of cash and credit purchases, computing unit costs and determining quantities to buy are all common applications of consumer mathematics. Many of the mathematical skills needed, such as the ability to compute, to use percents, to make measurements and to convert measurement units, have been discussed in previous chapters. The exercises presented here deal with situations that might be encountered by the consumer. All of the items are in word-problem format, so a person must determine the process to use before beginning calculations.

Also included in this chapter are exercises involving graphs and tables. Graphs and tables provide information on a wide variety of subjects, from the stock market to sports statistics. The alert consumer should be able to read and interpret such information.

The consumer-mathematics exercises were administered primarily to 17-year-old and adult respondents. First, the situations were not considered particularly relevant to 9- and 13-year-old concerns; second, many of the necessary mathematical skills are first introduced in the upper elementary grades and proficiency at ages 9 and 13 would not be expected.[1]

Consumer Situations

Several exercises required respondents to make cost comparisons. For example, in one item the cash price for an automobile ($2,850) and the credit price ($400 down and $80 per month for three years) were given and respondents had to determine how much more would be paid using credit. Fifty-six percent of the 17-year-olds and 68% of the adults correctly replied that the difference was $430.

Approximately one-half of the respondents at age 13, three-fourths at age 17 and close to 9 out of 10 of the adult respondents correctly stated the price difference between a 10% and a 15% discount on a $100 item. Fifty-six percent of the adults correctly figured the difference between a 3% and 4% sales tax for a car costing $2,760.00.

A typical supermarket cost-comparison item appears in Table 15. At all three ages (13, 17 and adult) the majority of respondents picked the largest size as having the lowest cost per ounce. Thirteen percent of the 13-year-olds and 10% of the 17-year-olds evidently thought that lowest price per ounce meant lowest price.

A similar exercise involved comparing unit costs for various-sized cans of tuna fish. After making a round estimate about which can would be the best buy, respondents were asked to give a specific cost-per-ounce estimate for each can. Forty percent of the 17-year-olds and 45% of the adults correctly determined the can that sold for the lowest cost per ounce, 46% at both ages 17 and adult incorrectly selected the largest can as selling

[1] Results for selected consumer-mathematics exercises are presented in this chapter. More detailed data on the consumer-mathematics exercises are found in the report *Consumer Math Selected Results From the First National Assessment of Mathematics, Report 04-MA-02* (Washington, D.C. Government Printing Office, 1975).

TABLE 15. Exercise and Results for Cost-Comparison Item

A housewife will pay the lowest price per ounce for rice if she buys it at the store which offers

	Age 13	Age 17	Adult
○ 12 ounces for 40 cents.	13%	10%	4%
○ 14 ounces for 45 cents.	9	8	5
● 1 pound, 12 ounces for 85 cents.	25	34	39
○ 2 pounds for 99 cents.	46	46	47
○ I don't know.	6	3	4
No response	2+	+*	+*

*Plus equals rounded percents less than one
+Figures may not add to 100% due to rounding error

for the lowest cost per ounce. Respondents were more successful in making specific unit-price estimates for each can. For 17-year-olds and adults, percentages of correct responses were 66% and 75% for the smallest can, 63% and 66% for the medium-sized can and 55% and 59% for the largest can. Figure 14 provides a comparison of results for the exercises discussed above.

Taxes

Taxes are a fact of life for most adult Americans. Although they cannot be avoided, everyone should be able to calculate his taxes so as not to pay more than his fair share.

National Assessment asked adults to use a portion of the federal income tax tables to determine the correct tax on a given income for a married couple claiming three exemptions. Fifty-five percent, or slightly over half, responded with the correct tax. About 10% used the table for two rather than three exemptions and another 13% responded with "I don't know." Although not all American adults use the federal tax tables, it appears somewhat surprising that 45% of the adults ages 26–35 have difficulty in reading tax tables intended for their use.

Thirteen-, 17-year-old and adult respondents used a sales tax table to find the tax on several different amounts. When asked to give the tax on an amount listed on the table, one-half of the 13-year-olds, three-fourths of the 17-year-olds and four-fifths of the adults were able to do so. However, when asked to give the tax for an amount higher than those listed on the table, 7% at age 13, 30% at age 17 and 60% at adult were successful.

A question on property taxes was answered correctly by 40% of the 17-year-olds and 56% of the adults. In this problem, the tax per $1,000 of assessed value and the assessed value of a house were given, and respondents determined what the tax would be. Figure 15 displays results for these tax problems.

Simulated Consumer Transactions

A number of assessment items were designed to simulate consumer situations more closely than could be done with paper-and-pencil items. For example, 9- and 13-year-olds were asked to make change for a purchase; 17-year-olds and adults were asked to balance a checkbook and fill out a time card.

The 9- and 13-year-olds were told to pretend that they were sales clerks and that the "customer" (the assessment administrator) wanted to buy a softball. Twenty-six percent of the 9-year-olds and 72% of the 13-year-olds returned the correct change from two dollars. (The price of the item was slightly over a dollar.)

Nearly one-third of the 17-year-olds and almost half the adults correctly calculated a

FIGURE 14. Results for Selected Exercises: Consumer Situations

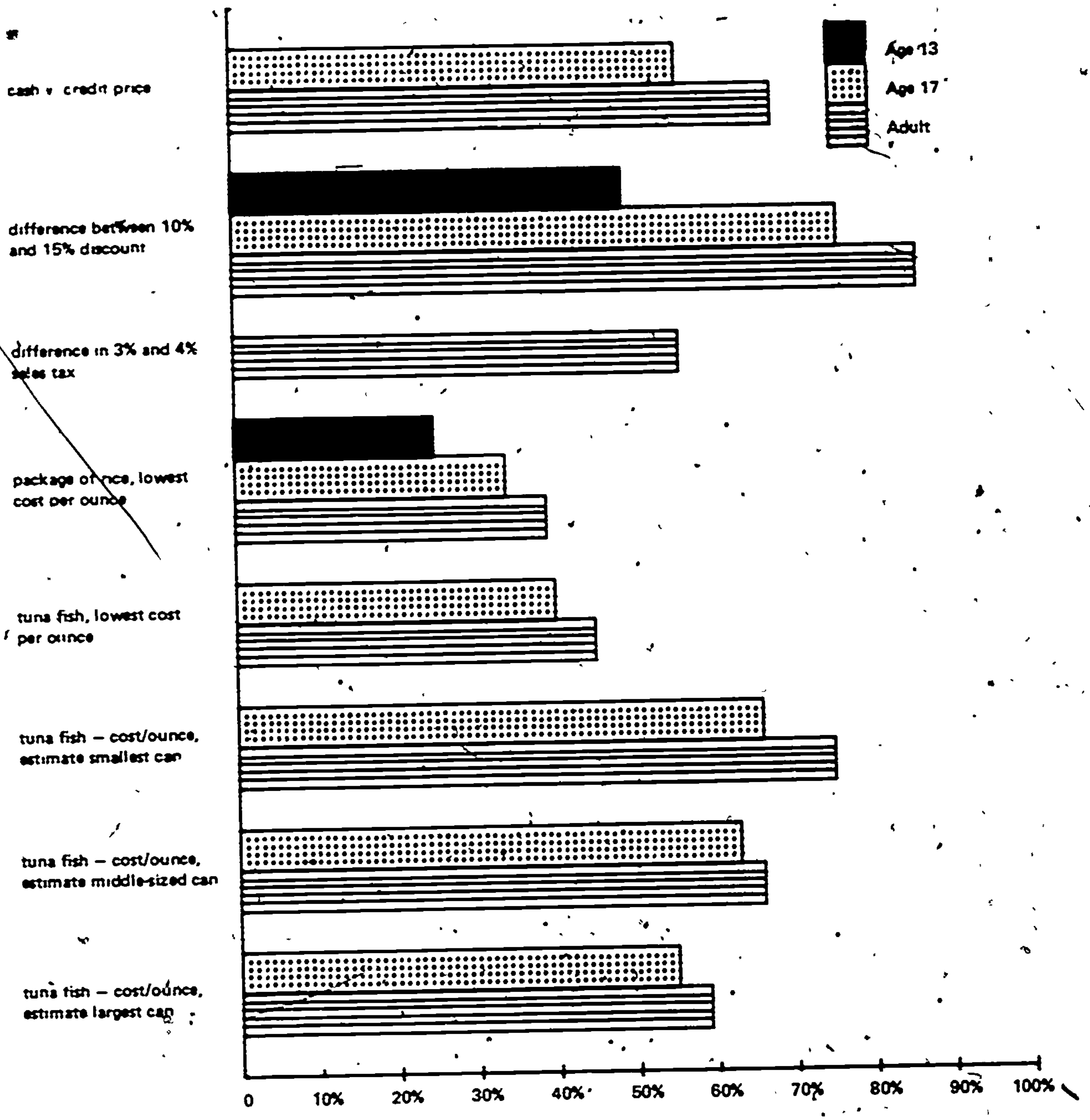

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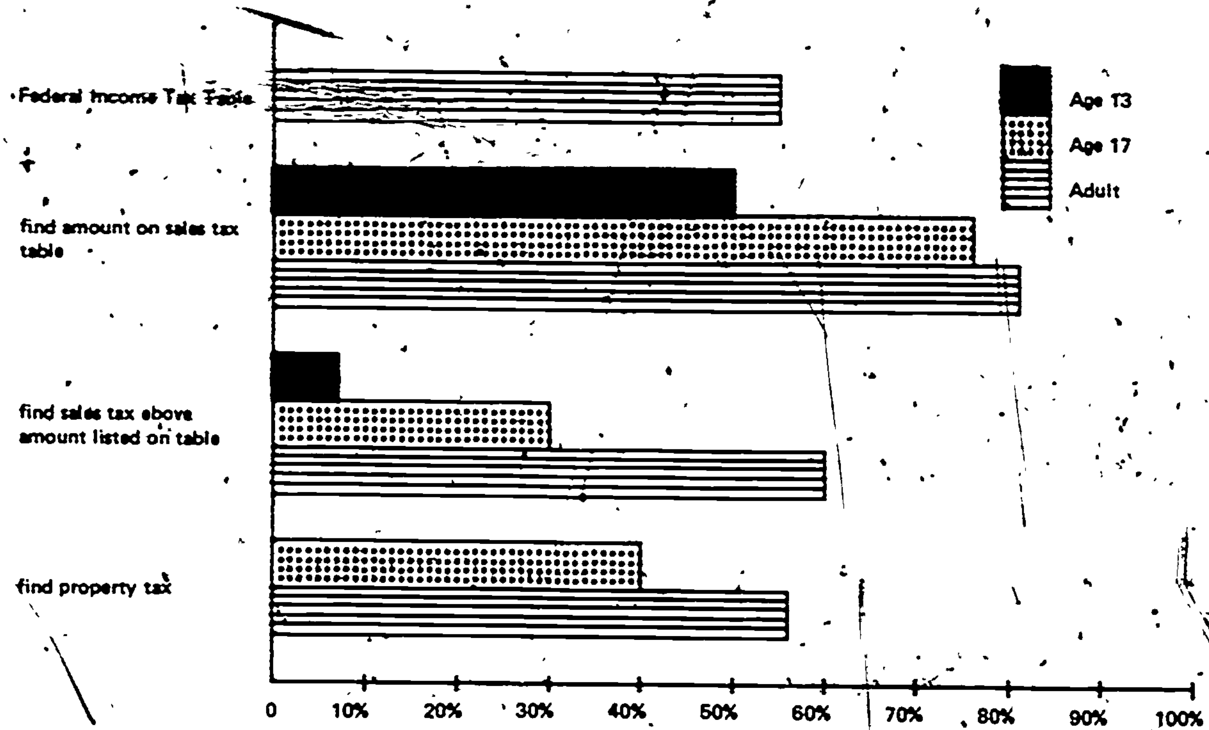
FIGURE 15. Results for Selected Exercises: Taxes



week's pay from a time card showing in and out times and an hourly rate of pay. An additional 48% at age 17 and 42% at adult used a correct procedure but made a mistake in calculation. People seemed to know how to use a time card but apparently needed to exercise greater care in completing computations.

Seventeen-year-old and adult respondents were asked to balance a checkbook. Each respondent received a bank statement, a check register and cancelled checks. In reconciling the statements, a subtraction error and a deposit error had to be corrected, and service charges and an outstanding check had to be included. Respondents were also asked whether they had ever had a checking account, and, if so, whether they had ever balanced a checkbook.

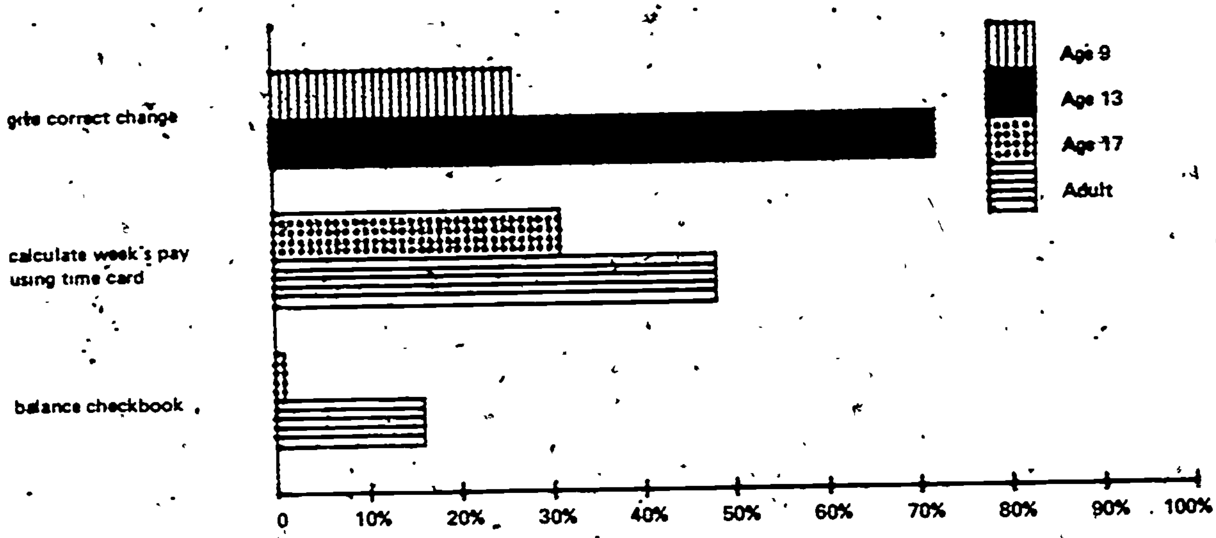
Sixteen percent of the 17-year-olds stated that they had a checking account at some time, but only 9% of the 17-year-olds had actually balanced their own account. Among the adults, 87% had or had had a checking account, and 72% stated that they had balanced a checkbook.

The small number of 17-year-olds having actual experience with balancing a checkbook undoubtedly accounts for their low percentages of success on this exercise: 1% of all 17-year-olds gave the correct balance. Sixteen percent of the adults balanced the checkbook — a substantial increase over the 17-year-olds but still not a large percentage considering the number who had at some time balanced a checkbook. Figure 16 shows performance levels on these three exercises.

Reading and Interpreting Graphs and Tables

Graphs and tables are often used in newspapers and popular magazines to simplify presentation of large amounts of data. These data may cover any subject — economic forecasts, medical research, sports statistics, to name a few. The consumer must understand how to read and interpret graphs and tables or the data will not be useful to him.

FIGURE 16. Results for Selected Exercises: Simulated Consumer Transactions



Bar graphs, pictographs (using symbols) and line graphs were included in the assessment exercises. Nine-year-olds read a bar graph showing weights of several children and chose the child weighing most, the child weighing least and the child weighing closest to a specified amount. Eighty-nine percent correctly identified the heaviest child, and 84% the lightest child. Finding the bar closest to a specific number appeared more difficult, with 61% of the 9-year-olds selecting the right child.

Thirteen-year-olds saw a pictograph that showed the rural populations of nine regions in the United States and were asked to name the two regions with the largest rural populations. Seventy-nine percent at age 13 correctly named the two regions; an additional 13% correctly named the one region with the largest rural population.

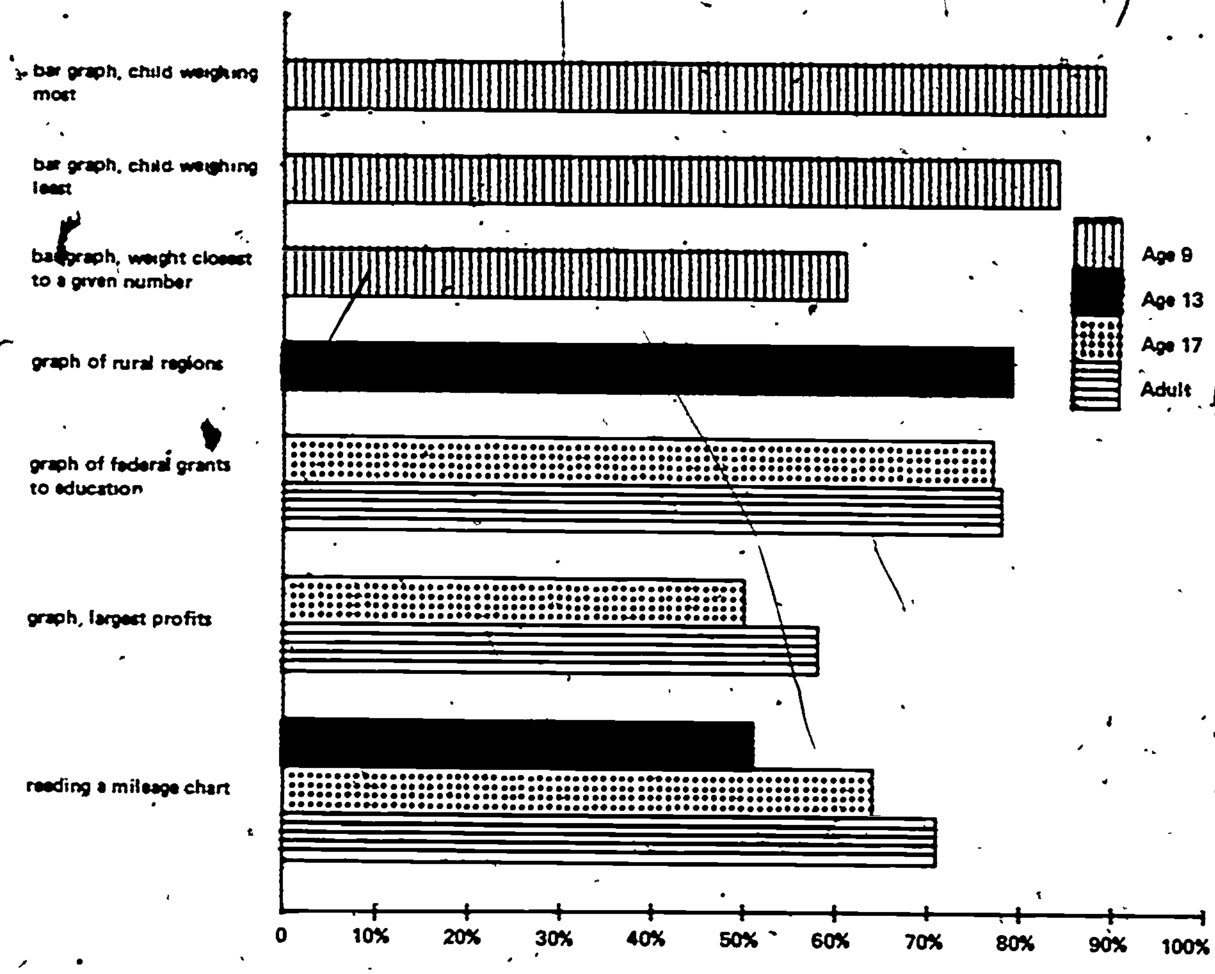
Respondents at ages 17 and adult used a pictograph depicting federal education grants for a five-year period to determine the year in which grants dropped. Seventy-seven percent of the 17-year-olds and 78% of the adults selected the correct year. The 17-year-olds and adults were less successful at determining the year in which the largest profits were made when shown a line graph displaying income and expenses for several years: 50% of the 17-year-olds and 58% of the adults chose the right year. The largest number of incorrect responses — 38% at age 17 and 24% at adult — were the year in which both income and expense were highest, indicating that some respondents may have had difficulty with the word profit.

Respondents at the upper three age levels used a mileage table, of the type found in road atlases, to find the distance between two cities. Fifty-one percent of the 13-year-olds, 64% of the 17-year-olds and 71% of the adults gave the correct distance. A fairly prevalent mistake, made by 15% of the 13-year-olds, 21% of the 17-year-olds and 16% of the adults, was to attempt to subtract the entries for the two cities in some manner. Results for these exercises on reading and interpreting graphs and tables appear in Figure 17.

Summary

It is difficult to summarize performance on the consumer-math exercises as the exercises were all quite different and demanded different types of skills. There was no one type of exercise upon which people did better or worse than others although percentages of

FIGURE 17. Results for Selected Exercises. Reading and Interpreting Graphs and Tables



success tended to be higher on exercises such as reading sales tax from a table and lower on those requiring computation with percents.

The following examples illustrate the range of success for each age level and also provide some indication of relative levels of performance across age levels. Thirteen-year-old results ranged from 49% to 54% on four exercises about reading sales tax from a table. One out of four 13-year-olds correctly determined which package of rice had the lowest cost per ounce, but under 1 in 12 (8%) figured out the money amount of a discount that was shown as a percent of a total amount.

At age 17, results on the sales-tax exercises went from 75% to 79%. Approximately one-third of the 17-year-olds correctly chose the rice package with the lowest price per ounce and 36% were able to give the money amount of the discount.

Adult percentages of success went from 78% to 82% on the sales-tax exercises. Thirty-nine percent of the adults selected the rice package with the lowest unit cost, and 66% (two-thirds) gave the correct amount of discount.

Seventeen-year-old performance was above that of 13-year-olds, and adult performance was higher than that of 17-year-olds on the majority of the consumer-math items. Median percentages and overlap median percentages are shown in Table 16.

TABLE 16. Median and Overlap Median Percentages of Success – Consumer-Mathematics Exercises

	Age 13	Age 17	Adult
Median percentages of success	49%	54%	62%
Number of exercises summarized	(14)	(34)	(41)

	Age 13 – Age 17		Age 17 – Adult	
Overlap median percentages of success	47%	66%	54%	61%
Number of exercises summarized	(12)	(12)	(34)	(34)

CHAPTER 7

SUMMARY

We have seen how young Americans perform in various branches of mathematics. How do we evaluate their abilities? First, we can examine their performances in the different content areas. Second, we can observe the results for the various population groups, noting which groups consistently deviate from national levels and the extent of their deviation. To determine the branches of mathematics in which people experienced the greatest and least success, the six content areas were ranked by median percentages of success. At all ages, people did best on measurement and numbers and numeration. Relative performance on each of the content areas is shown in Table 17.

Nine-year-olds, 13-year-olds and 17-year-olds showed the same relative performance on all the content areas, with the single exception that 17-year-olds had greater success on numbers and numeration. The pattern of results was similar for adults although their performance in consumer math was slightly better than that for geometry. A caution is in order. Again, sets of exercises are not the same and thus, the medians are not absolute numbers that can be compared. The numbers are meaningful *only* insofar as the reader recognizes and takes cognizance of their limitations.

What do these numbers mean? They simply mean that on the exercises the National Assessment of Educational Progress (NAEP) designated as "measurement," people did considerably better than on exercises designated as "variables and relationships." The

TABLE 17. Rank Order of Median National Percentages by Content Area

	Age 9	Age 13	Age 17	Adult
Measurement	46%	63%	65%	73%
Number of exercises summarized	(35)	(35)	(29)	(29)
Numbers and numeration	38	60	70	65
Number of exercises summarized	(74)	(86)	(74)	(44)
Geometry	28	51	57	53
Number of exercises summarized	(39)	(37)	(37)	(29)
Consumer math	*	49	54	62
Number of exercises summarized		(14)	(34)	(41)
Variables and relationships	*	39	38	49
Number of exercises summarized		(28)	(50)	(24)
Probability and statistics	*	*	27	32
Number of exercises summarized			(17)	(12)

**There were not enough exercises in this content area for a meaningful summary.*

exercises chosen by NAEP represent the content area to the best of our ability; however, they may not be in accord with the reader's concept of the content area.

In addition to comparing results for content areas, we can make age-level comparisons using overlap medians to examine performance at consecutive age levels on identical exercises (see Table 18).

From ages 9 to 13 and 13 to 17 there was, as would be expected, an increase in performance at the older age levels. The greatest increase from age 9 to 13 was in the numbers and numeration content area. From age 13 to 17, the greatest jump was in the section dealing with variables and relationships. The difference in performance between 17-year-olds and adults was not as consistent. Adults held an advantage on measurement, consumer math and probability and statistics. Seventeen-year-olds showed the greatest advantage in variables and relationships, they also outperformed adults in geometry and in numbers and numeration.

Reporting Groups

In addition to reporting national performance levels, National Assessment provides results for various groups within the national population. Results are reported for sex, race, region of the country, size and type of community and level of parental education. The differences in achievement among these groups provide an indication of areas of strength and weakness in American education.

The National Assessment reporting groups are defined as follows:

Sex

Results are presented for males and females.

Race

Currently, results are reported for blacks and whites.

TABLE 18. Overlap Median National Percentages by Content Area

	Age 9	Age 13	Age 13	Age 17	Age 17	Adult
Numbers and numeration	29%	80%	51%	72%	69%	63%
Number of exercises summarized	(43)	(43)	(67)	(67)	(43)	(43)
Measurement	45	73	59	78	65	73
Number of exercises summarized	(22)	(22)	(19)	(19)	(29)	(29)
Geometry	28	58	47	70	60	53
Number of exercises summarized	(27)	(27)	(29)	(29)	(29)	(29)
Consumer math	*	*	47	66	54	61
Number of exercises summarized			(12)	(12)	(34)	(34)
Variables and relationships	*	*	39	67	64	49
Number of exercises summarized			(24)	(24)	(21)	(21)
Probability and statistics	*	*	*	*	29	32
Number of exercises summarized					(12)	(12)

Number of exercises was not sufficient for summary purposes.

Region

The country has been divided into four regions — Southeast, West, Central and Northeast. The states that are included in each region are shown in Exhibit 3.

Parental Education

Four categories of parental education are defined by National Assessment. These categories include: (1) those whose parents have had no high school education, (2) those who have at least one parent with some high school education, (3) those who have at least one parent who graduated from high school and (4) those who have at least one parent who has had some post-high school education.

Size and Type of Community

Community types are identified both by the size of the community and by the type of employment of the majority of people in the community.

High metro. Areas in or around cities with a population greater than 200,000 where a high proportion of the residents are in professional or managerial positions.

Low metro. Areas in or around cities with a population greater than 200,000 where a high proportion of the residents are on welfare or are not regularly employed.

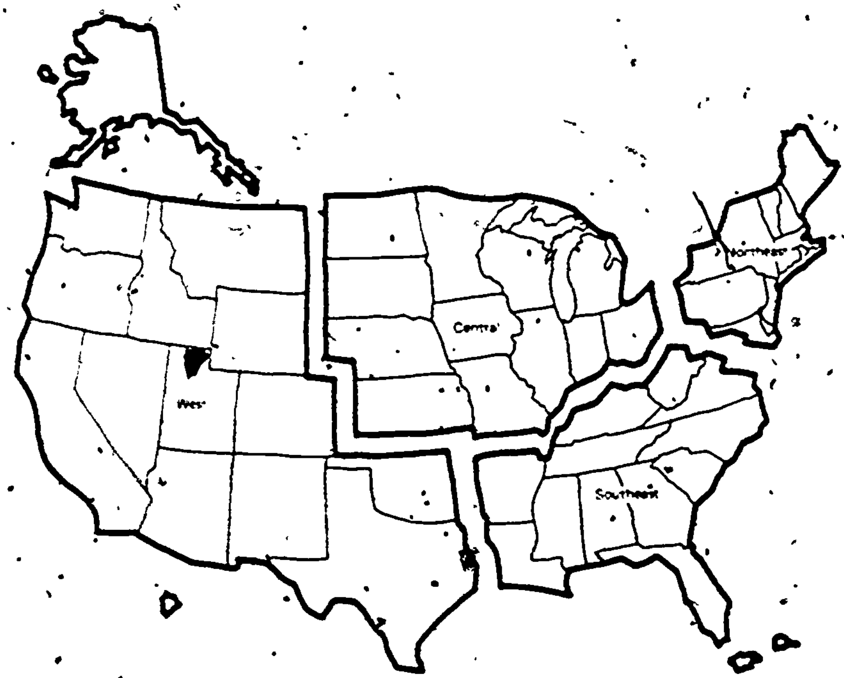
Extreme rural. Areas with a population under 10,000 where most of the residents are farmers or farm workers.

Urban fringe. Communities within the metropolitan area of a city with a population greater than 200,000, outside the city limits and not in the high- or low-metro groups.

Main big city. Communities within the city limits of a city with a population over 200,000 and not included in the high- or low-metro groups.

Medium city. Cities with populations between 25,000 and 200,000.

EXHIBIT 3. National Assessment Geographic Regions



Small places. Communities with a population of less than 25,000 and not in the extreme-rural group.

Results for each of the reporting groups are discussed in terms of difference from the national performance. Thus, some groups will naturally be above the national level and others below. However, if certain groups consistently perform above or below the nation, this may be evidence of educational disparities, which might bear further investigation.

Group Differences From the National Percentage

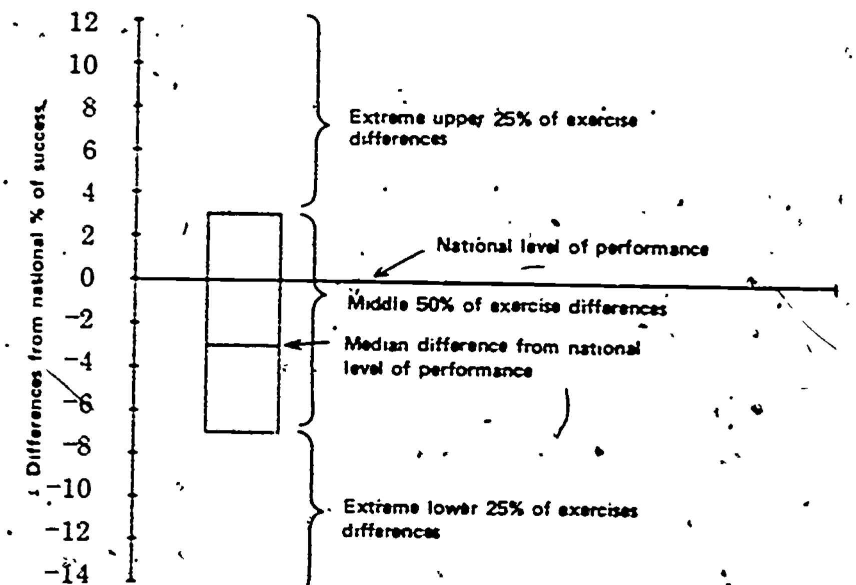
In a given set of exercises (e.g., all mathematics exercises), a group's achievement can be summarized conveniently by examining its differences from national percentages of success. If a group's percentage is lower than the national percentage, the difference is expressed as a negative number; otherwise, it is a positive number. For example, if 70% of the nation's 9-year-olds correctly answer a given exercise while 73% of the 9-year-olds in the Northeast correctly answer the same exercise, the Northeast difference is +3.

If we wish to discuss Northeast performance across a number of exercises, the single most useful number is the median difference. However, a more complete picture of a group's typical performance emerges from examination of the entire range of differences or, more conveniently, the middle 50% of the exercise differences. The group summary exhibits in this chapter depict not only the median differences for each group, but the range including the middle 50% of the exercise differences as well.

Since different sets of exercises were presented to individuals of different ages, the number of exercises or exercise parts for which group differences have been computed varies across the four age levels.

Exhibit 4 presents a sample plot of group differences at age 9. Differences on all exercises included range from +12 to -14, but since only the middle half of the differences are shown, the bar is plotted from only +3 to -7. The median difference, -3, is indicated by the horizontal line crossing each bar.

EXHIBIT 4. Sample Graph. Differences From National Percentages of Success



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Group Performance

Sex

Male and female mathematical abilities appeared much the same at ages 9 and 13, but by ages 17 and adult, males exhibited a definite advantage. This advantage increased in all content areas from age 17 to adult. Table 19 presents the median difference of male and female percentages from national percentages for each of the content areas and for all the mathematics exercises. At ages 9, 13 and 17, males showed the greatest advantage over females on the measurement and geometry content areas while at the adult level, the divergence in male and female performance was largest for the geometry and for the variables and relationships content areas.

Figure 18 displays the median difference and the range of differences for males and females on all the mathematics exercises at each age level.

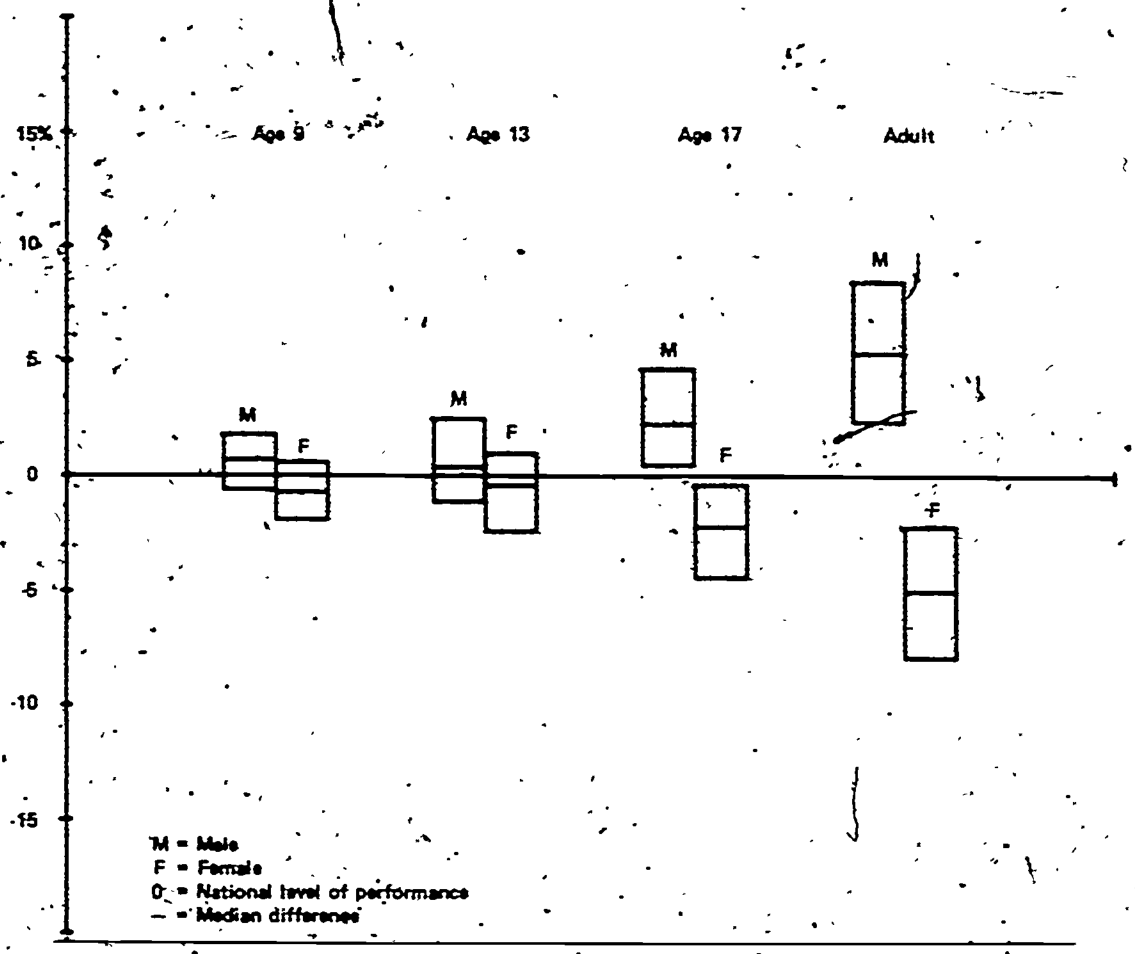
Although the generalization does not hold true in every case, males and females tended

TABLE 19. Male-Female Median Differences From National Performance by Content Area

	Age 9	Age 13	Age 17	Adult
Overall				
Male	0.7	0.4	2.3	5.4
Female	-0.7	-0.4	-2.2	-5.0
Number of exercises summarized	(163)	(211)	(245)	(179)
Numbers and numeration				
Male	-0.6	-0.5	1.5	4.4
Female	0.6	0.4	-1.4	-4.1
Number of exercises summarized	(74)	(86)	(74)	(44)
Measurement				
Male	1.7	2.1	4.2	4.1
Female	-1.6	-2.2	-3.6	-3.8
Number of exercises summarized	(35)	(35)	(29)	(29)
Geometry				
Male	1.0	1.5	4.8	6.5
Female	-1.0	-1.4	-4.6	-6.1
Number of exercises summarized	(39)	(37)	(37)	(29)
Variables and relationships				
Male	*	-0.2	1.2	6.4
Female		0.2	-1.2	-6.0
Number of exercises summarized		(28)	(50)	(21)
Probability and statistics				
Male	*	*	1.5	4.9
Female			-1.5	-4.6
Number of exercises summarized			(17)	(12)
Consumer math				
Male	*	0.2	2.6	4.8
Female		-0.2	-2.6	-4.4
Number of exercises summarized		(14)	(34)	(41)

**There were not enough exercises in this content area for a meaningful summary.*

FIGURE 18. Differences From National Performance on all Math Exercises by Sex



to have facility with different types of exercises. Females were best at completing strictly computational exercises in which the operation to be used was given in the problem. At the upper age levels, this advantage extended to calculations with rational numbers. Males, on the other hand, showed the greatest advantage at ages 9 and 13 in using a ruler and a thermometer at the smaller intervals and in making measurement conversions. The male advantage at ages 17 and adult was most marked on exercises about using metric units, using a protractor to measure angles and figuring area and volume.

Race

Blacks were at a disadvantage in all areas of the mathematics assessment, a disadvantage that was more marked at the upper age levels. The difference in black and white performance was greatest at ages 9, 13 and 17 on the measurement section, the section in which national median percentages were, in general, highest (see Table 17). At the 13-year-old and adult levels, the differences for the consumer-mathematics content areas were quite similar to those for the measurement content area. Table 20 gives median differences in group performance for blacks and whites in each content area for each age level. It should be remembered that, although the difference in black and white performance is least on the probability and statistics exercises, national performance on these exercises was lower, restricting the size of possible differences.

Figure 19 displays the median differences and distribution of the black and white group

TABLE 20. Black-White Median Differences From National Performance by Content Area

	Age 9	Age 13	Age 17	Adult
Overall				
Black	-12.9	-18.5	-21.0	-24.5
White	2.9	3.9	4.0	3.9
Number of exercises summarized	(163)	(211)	(245)	(179)
Numbers and numeration				
Black	-14.9	-17.9	-20.8	-22.8
White	3.3	3.7	4.3	3.2
Number of exercises summarized	(74)	(86)	(74)	(44)
Measurement				
Black	-17.0	-22.2	-24.3	-25.4
White	3.6	4.1	4.9	4.3
Number of exercises summarized	(35)	(35)	(29)	(29)
Geometry				
Black	-5.6	-19.8	-22.6	-23.7
White	1.3	3.9	4.0	4.0
Number of exercises summarized	(39)	(37)	(37)	(29)
Variables and relationships	*			
Black		-18.3	-17.5	-22.6
White		4.0	3.4	3.4
Number of exercises summarized		(28)	(50)	(21)
Probability and statistics	*	*		
Black			-12.7	-16.5
White			2.4	2.7
Number of exercises summarized			(17)	(12)
Consumer math	*			
Black		-21.0	-22.3	-25.6
White		5.2	4.0	4.2
Number of exercises summarized		(14)	(34)	(41)

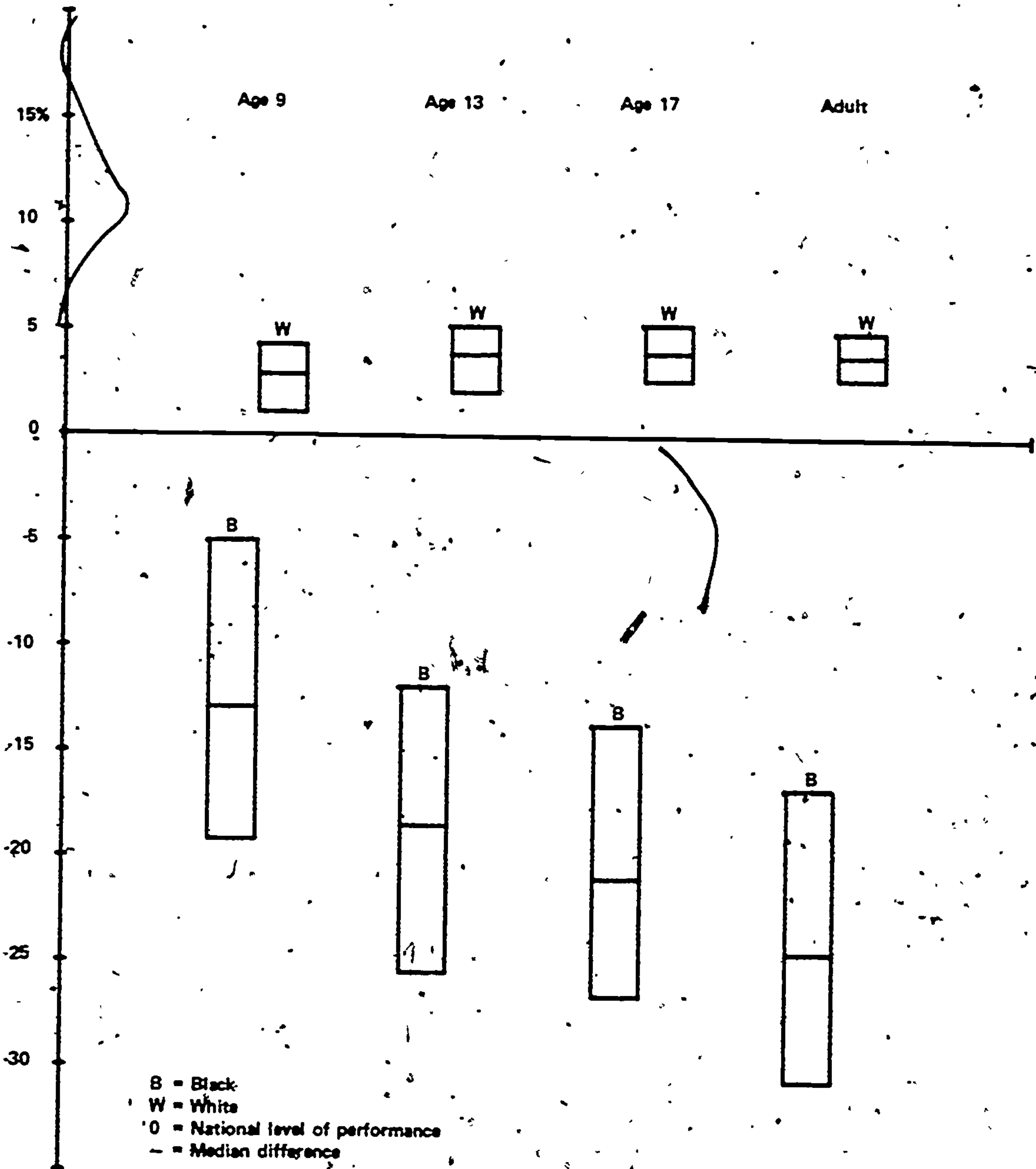
There were not enough exercises in this content area for a meaningful summary.

differences from national performance for each exercise in the mathematics assessment.

The types of exercises that each group did well and poorly on are not so clearly defined as in the male-female differences; however, some observations can be made. Black performance tended to be closest to that of the nation when national percentages were very low or very high (i.e., very easy or very hard exercises). For example, black 13-year-olds were fairly successful (3 to 5 percentage points below whites) on such things as identifying a circle, adding 3 + 0 and adding 38 + 19. On exercises such as identifying an ellipse or converting from degrees Fahrenheit to degrees Centigrade, the national 13-year-old level of performance was under 5%. Thus, on such exercises the black deficit could not be more than 5 percentage points.

Black 9-year-olds performed furthest below the nation on identifying half inches marked on a ruler and on drawing hands on a clock to represent a specific time; black 13-year-olds showed large (over 30-percentage-point) deficits on exercises about setting a thermometer, identifying 3/4 inch marked on a ruler and

FIGURE 19. Differences From National Performance on all Math Exercises by Race



giving geometric names for common shapes. Black 17-year-olds and adults had considerable difficulty compared to whites in giving geometric names for common shapes, converting feet to yards, figuring square feet and using map scales.

Region

Patterns of difference in regional performance shifted somewhat with the age of the respondents. Performance in the Southeast was below that of the nation at all age levels in the

mathematics assessment (as it has been in most learning areas assessed by National Assessment[1]), but the difference was smaller at the adult level than for the school-age groups. In contrast, performance in the Northeast was above the nation at ages 9, 13 and 17 but became very close to the national level for adults.

The difference in school-age and adult performance was not as noticeable in the Central and West regions. Adults in the Central region showed a smaller advantage above the nation than did their 9-, 13- and 17-year-old counterparts; Western adults performed above the national level while 9-, 13- and 17-year-olds living in the West did not. Figure 20 displays the median difference and the range of differences from national performance on all mathematics exercises for each region at each age level.

Parental Education

National Assessment uses the level of education attained by one's parents as one form of socioeconomic measure. Results from previous assessments have indicated that the education level of one's parents is related to one's own academic achievement.

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[1] *Update on Education A Digest of the National Assessment of Educational Progress* (Denver, Colo.: Education Commission of the States, 1975).

FIGURE 20. Differences From National Performance on all Math Exercises by Region

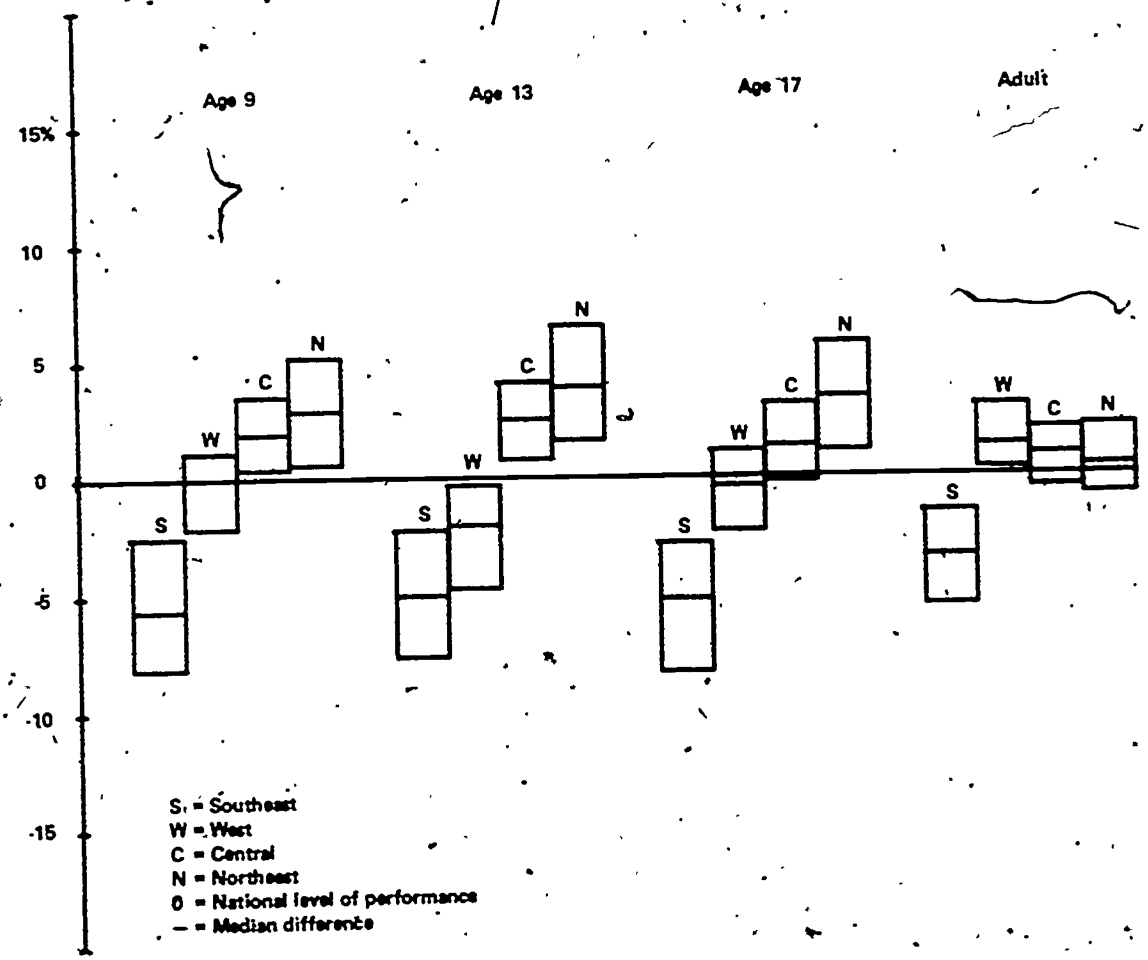
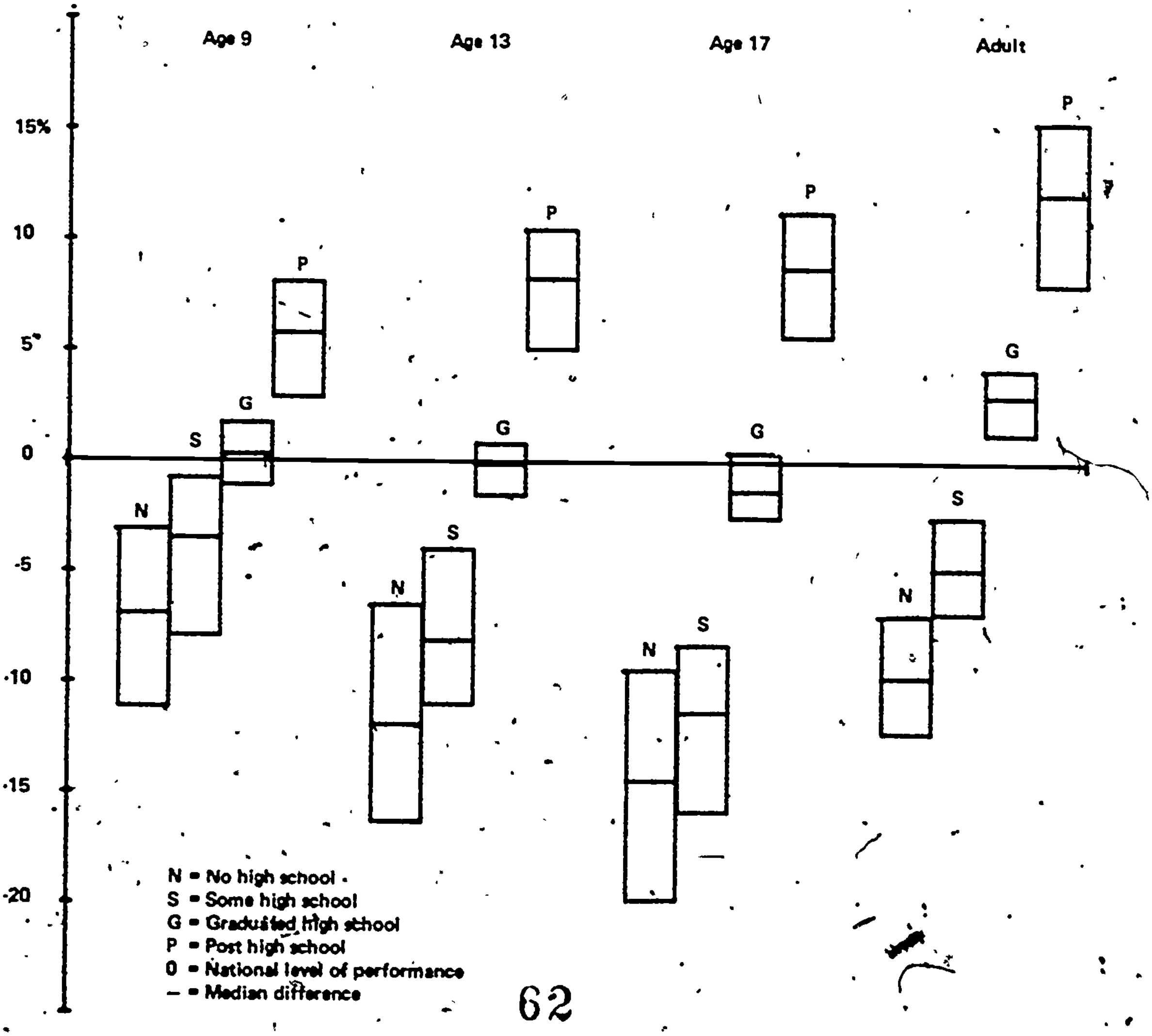


Figure 21 shows median differences and the range of differences in performance on all mathematics exercises for each parental-education group at each age level. At all ages, performance was below that of the nation for both those with neither parent having any high school education and for those with one or both parents having some high school education without having graduated. Those with at least one parent who graduated from high school performed very close to national levels and those with at least one parent having some type of post-high school education performed consistently above national levels.

School-age respondents in the no-high-school-education and some-high-school-education groups dropped further below the nation at ages 13 and 17. A similar pattern was evident for the graduated-high-school group. Performance of the post-high-school group, which was above that of the nation at all age levels, showed an increase from age 9 to 13 and a fairly consistent level from age 13 to 17.

Results for adults differed somewhat from the pattern for 9-, 13- and 17-year-olds. Adults in the two lower parental-education groups generally were not as far below the national level as their 13- and 17-year-old counterparts. The

FIGURE 21. Differences From National Performance on all Math Exercises by Parental Education



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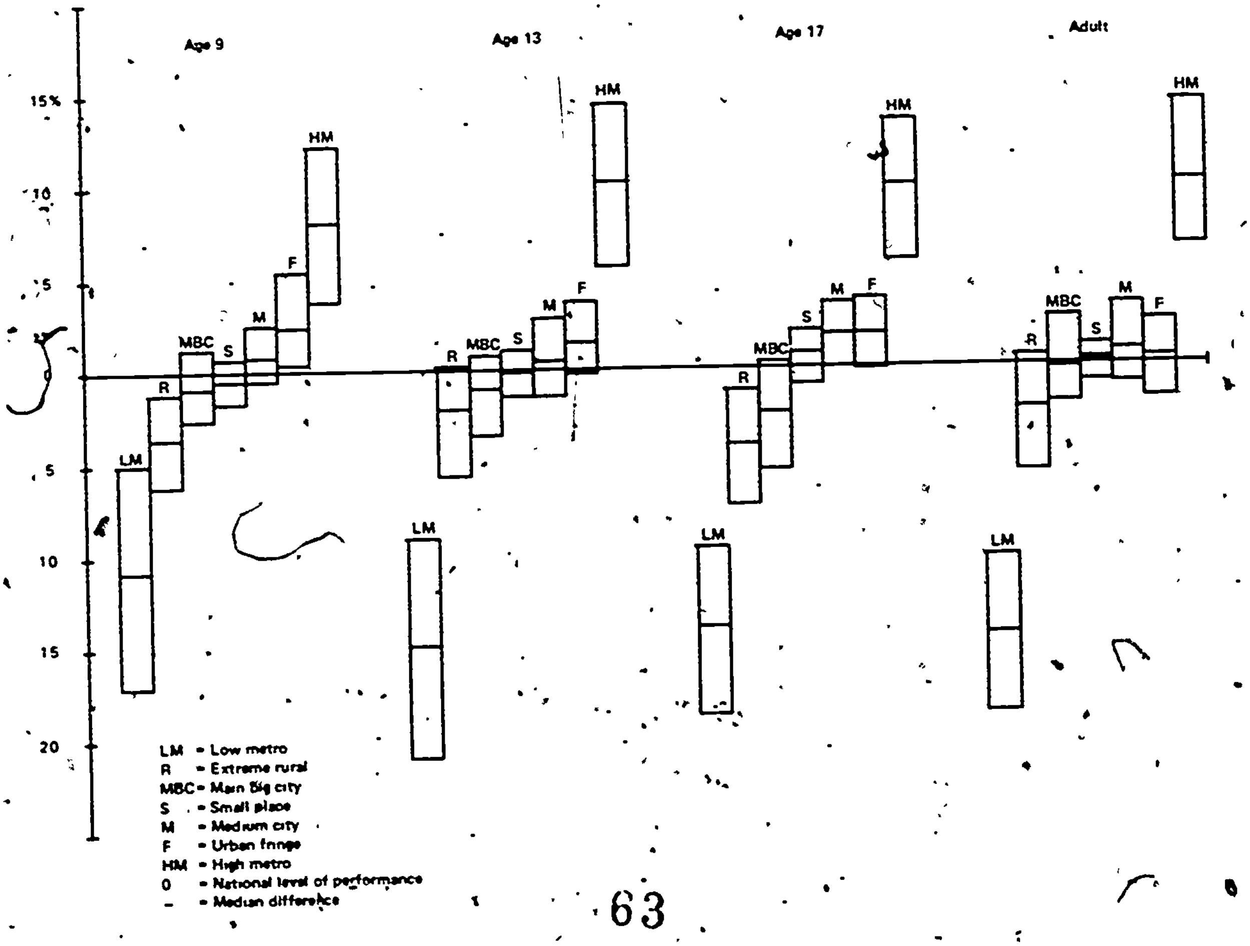
graduated- and post-high-school adults showed a better performance relative to the nation than did 13- and 17-year-olds in these groups.

Size and Type of Community

As another indicator of respondents' backgrounds, National Assessment identifies seven different community types. Three extreme types of community — high metropolitan, low metropolitan and extreme rural — are defined by size of community and type of occupation of the majority of people in the community. (See the section entitled Reporting Groups, page 42, for complete definitions.)

Figure 22 presents the median difference and the range of differences in performance for each of the community groups at each age level. The high-metropolitan respondents performed above the nation at all age levels; however, the advantage was not as marked for 9-year-olds. The advantage above the nation for the high-metro respondents was virtually identical at the three upper age levels. The low-metro group results were substantially below those for the nation at all age levels. Median differences for 13-, 17-year-olds and adults were quite similar, but the deficit for the low-metro 9-year-olds was smaller than for the other age levels. Results for the urban fringe were consistently above those for the nation, while those for the extreme-rural and

FIGURE 22. Differences From National Performance on all Math Exercises by Size and Type of Community



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the main-big-city groups were consistently below.

Conclusion

In presenting data from the mathematics assessment, National Assessment described the status of the mathematical abilities of certain groups of people at a particular point in time. The data describe the conditions that existed; they do not supply value judgments regarding those conditions.

After surveying the data, we can make some generalizations about American mathematical abilities. People tended to do best on skills such as ordering numbers, completing computations and making measurements. They showed less ability in the areas of variables and relationships and probability and statistics. This raises the following questions: How much mathematics does the average citizen need to function adequately in today's world? Are variables and relationships or probability and statistics that relevant to the ordinary person's needs? Such questions immediately generate other questions. Who is the "average citizen"? What does "function adequately" involve? These are philosophical issues that can only be answered in light of each reader's background and experience. However, such questions should be kept in mind when evaluating the implications of National Assessment data.

The disparities in group performance also raise questions to be investigated. As in other learning areas assessed by National Assessment, blacks, persons living in the Southeast, persons whose parents had little or no high school education and persons living in low-metropolitan communities performed consistently below the nation as a whole. Are these differences in performance inevitable? The differences in performance between the various regional and parental-education groups decreased at the adult level, however, for blacks and females the differences increased for the 17-year-old and adult respondents. Are the schools acting to change or to reinforce established patterns of group performance?

National Assessment hopes that readers will use the data compiled in the mathematics assessment to ask additional questions about the effectiveness of mathematics education and to institute research into factors influencing achievement. Data on present ability levels can be used to monitor changes in mathematics performance and to establish a reference point from which to chart future directions in American education.

U. S. GOVERNMENT PRINTING OFFICE 1975 683 773/209